[Ampère's law] .quiz

1) solenoid and Ampère's law

A solenoid with a length L and a radius R, and four different paths A, B, C, D are shown in fig.1. The current passing through the solenoid is I, with direction shown in the figure. We know that the magnetic field at the center is,

$$B_{\text{center}} = \mu_0 NI/\sqrt{4R^2 + L^2}. \tag{1}$$

a) Find $\oint_A \mathbf{B} \cdot \mathrm{d}\mathbf{l}$.
b) Find $\oint_B \mathbf{B} \cdot \mathrm{d}\mathbf{l}$.
c) Find $\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l}$.
d) When $L \ll R$ solenoid becomes a ring. Find the magnetic field of this ring at the center using (1).
e) Using (1) show that for a really long solenoid, i.e. $R \ll L$, the magnetic field at the center can be written as $B = \mu_0 nI$ where $n = N/L$.
f) The path D is an infinite line passing through the solenoid as shown in fig. 1. Find $\int_D \mathbf{B}.\mathrm{d}\mathbf{l}$.

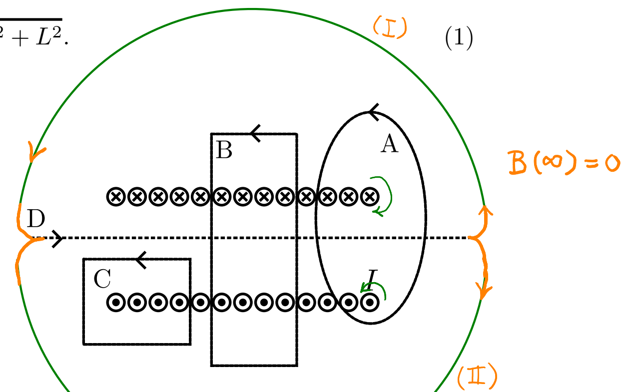

Figure 1: Solenoid and Ampère's law. The loops A, B, and C are on the plane that cuts the solenoid in half.

a) $\oint_A \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed by A}} = \mu_0 (2I - 3I) = -\mu_0 I.$

b) $\oint_B \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc. by B}} = \mu_0 (4I - 4I) = 0.$

c) $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc. by C}} = 4\mu_0 I.$

d) $R \gg L \rightarrow B_{\text{center}} = \dfrac{\mu_0 NI}{\sqrt{4R^2 + L^2}} = \dfrac{\mu_0 NI}{2R\sqrt{1 + \left(\frac{L}{2R}\right)^2}} = \dfrac{\mu_0 NI}{2R} + O\left(\left[\frac{L}{R}\right]^2\right).$

So for a ring with N turns, in other words, a circular coil, we have, $B_{\text{center}} = \dfrac{\mu_0 NI}{2R}.$

e) $R \ll L \rightarrow B_{\text{center}} = \dfrac{\mu_0 NI}{L\sqrt{1 + \left(\frac{2R}{L}\right)^2}} = \dfrac{\mu_0 NI}{L} + O\left(\left[\frac{R}{L}\right]^2\right) = \mu_0 nI + O\left(\left[\frac{R}{L}\right]^2\right).$

So for a really long solenoid, $B = \mu_0 nI$. One can prove that it's true for any point inside a long solenoid not only the axis.

f) One can close D at infinity, using either path (I) or (II) as shown in the fig. 1.

$$\int_D \vec{B} \cdot d\vec{l} = \oint_{D+(I)} \vec{B} \cdot d\vec{l} - \underbrace{\int_{(I)} \cancelto{0}{\vec{B}} \cdot d\vec{l}}_{B(\infty) = 0} = \oint_{D+(I)} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc. by } D+(I)} = -\mu_0 NI.$$

there is no magnetic field far from the solenoid