tutorial #14 [nuclear physics and radioactivity] .quiz

Same Z

1) Two isotopes of a certain element have binding energies that differ by 5.03 MeV. The isotope with the larger binding energy contains one more neutron than the other isotope. Find the difference in atomic mass between the two isotopes.

binding energy $= (\Delta m) c^2$ for ${}^{A}_{Z}X$: $\Delta m = Z m_p + \underbrace{N}_{A-Z} m_n -$ atomic mass of ${}^{A}_{Z}X$

$(\Delta m_1 - \Delta m_2) c^2 = 5.03$ MeV & $\begin{cases} \Delta m_1 = Z m_p + (N+1) m_n - M_1 \\ \Delta m_2 = Z m_p + N m_n - M_2 \end{cases}$

$\rightarrow (m_n - M_1 + M_2) c^2 = 5.03 \text{ MeV} \rightarrow M_1 - M_2 = m_n - (5.03 \text{ MeV})/c^2 = 1.008665\ u - \frac{5.03 \text{ MeV}}{931.5 \text{ MeV}} u$

$= 1.003265\ u$

2) Radon ${}^{220}_{86}$Rn produces a daughter nucleus that is radioactive. The daughter, in turn, produces its own radioactive daughter, and so on. This process continues until lead ${}^{208}_{82}$Pb is reached. What are the total number N_α of α particles and the total number N_β of β^- particles that are generated in this series of radioactive decays? Assume that there is no β^+ particle generated.

$${}^{220}_{86}\text{Rn} \xrightarrow[\substack{= \\ (ii)}]{\substack{(i) \\ =}} {}^{208}_{82}\text{Pb} + N_\alpha\, {}^{4}_{2}\text{He} + N_\beta \left({}^{0}_{-1}e\right)$$

$$\begin{cases} \#\text{ of } p+n: & 220 = 208 + 4N_\alpha + 0\,N_\beta \quad (i) \\ \#\text{ of } p: & 86 = 82 + 2N_\alpha - N_\beta \quad (ii) \end{cases}$$

$(i) \rightarrow N_\alpha = 3 \xrightarrow{(ii)} N_\beta = 2$.

3) The number of radioactive nuclei present at the start of an experiment is 4.60×10^{15}. The number present twenty days later is 8.14×10^{14}. What is the half-life (in days) of the nuclei?

$N_t = N_0 e^{-\lambda t}$

$N_0 = 4.60 \times 10^{15}$

t = 20 days : $N = 8.14 \times 10^{14} = N_0 e^{-\lambda \cdot 20 \text{ days}} \rightarrow \ln \frac{N}{N_0} = -\lambda \cdot 20 \text{ days} \rightarrow \lambda = \frac{1}{20 \text{ days}} \ln \frac{N_0}{N}$

$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \ln 2 \cdot 20 \text{ days} \cdot \frac{1}{\ln \frac{N_0}{N}} = 8.0$ days .