tutorial #4 [electric circuits (cont.)] .quiz

1) Ewald Georg von Kleist connects two $\mathcal{E} = 12$ V batteries with some $r = 100\ \Omega$ resistors and a capacitor on top with capacitance $C = 1.0\ \mu$F. He wants to find out how fast the capacitor charges, and at the end how much charge the capacitor would have. Ignore the capacitor for the first three parts.

a) We name the currents on the left and right branches I_1 and I_2. What is the current passing through the middle branch in terms of I_1 and I_2?
b) Now that you know all the currents in terms of I_1 and I_2, write two equations for voltages around the left and right loops. (Kirchhoff's loop rules)
c) Solve these equations and find I_1 and I_2. Now calculate $V_\mathrm{A} - V_\mathrm{B}$, which is the voltage the capacitor will have after a few time constants.
d) Find the final charge on the capacitor.
e) Now draw the same circuit, with the capacitor, but short circuit the batteries. Find the equivalent resistor from the capacitor's terminals.
f) What is the time constant for charging the capacitor?

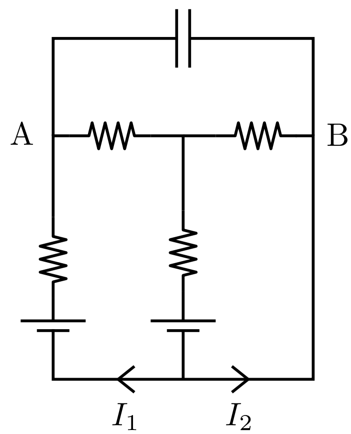


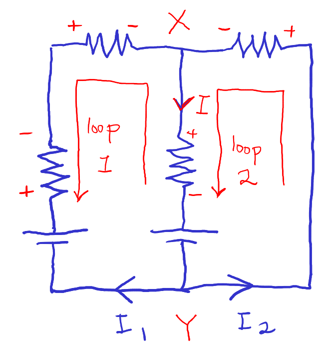
Figure 1: The circuit.

a) junction rule at X or Y : $I - I_1 - I_2 = 0 \rightarrow I = I_1 + I_2$.

b) loop 1 : $\mathcal{E} + r(I_1 + I_2) + rI_1 + rI_1 - \mathcal{E} = 0$

loop 2 : $-rI_2 - r(I_1 + I_2) - \mathcal{E} = 0$

Simplified : $\begin{cases} 3I_1 + I_2 = 0 & \text{(i)} \\ 2rI_2 + rI_1 = -\mathcal{E} & \text{(ii)} \end{cases}$



c) (i) $\Rightarrow$ $I_2 = -3I_1$ Substituting in (ii)

$\Rightarrow -6rI_1 + rI_1 = -\mathcal{E}$

$\Rightarrow I_1 = \frac{\mathcal{E}}{5r} = 24$ mA

and $I_2 = -3I_1 = -72$ mA

the negative sign means that the real current is in opposite direction.

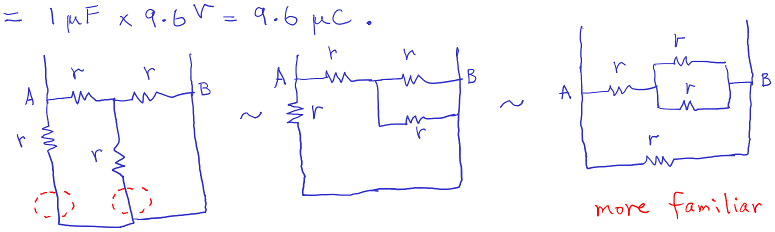
$V_A - V_B = rI_1 - rI_2 = 100^{\Omega} \times 24^{mA} - 100^{\Omega} \times (-72^{mA}) = 9.6^{V}$.

d) $Q = CV = C(V_A - V_B) = 1\,\mu F \times 9.6^{V} = 9.6\ \mu C$.

e) $R_{eq} = r \parallel [r + (r \parallel r)]$

$= \frac{3}{5} r = 60^{\Omega}$

batteries short-circuited



f) $\tau = R_{eq} C = 60^{\Omega} \times 1^{\mu F} = 60^{\mu s}$.