1) Joseph Henry puts a sliding rod with length $L = 10$ cm and weight $mg = 1.0$ N on a circuit as shown in fig. 1 with resistance $R = 10\ \Omega$. The rod is sliding down the tracks with no friction and the resistance of vertical tracks and rod is negligible. There is a uniform magnetic field, $B = 1.0$ T perpendicular to the plane as shown in the figure. Call the velocity of the rod, v, going down. In the first three parts find the answer in terms of v.

a) Find the emf induced in the circuit.
b) Find the current passing through the circuit. Draw the direction.
c) What is the magnetic force on the rod?
d) Now if v remains constant (zero acceleration), what is the value of $v = v_c$?
e) Joseph stops the rod, and release it again (initial velocity = 0). So the rod starts accelerating. Would rod accelerate forever? Why?
f) Joseph pushes the rod so initially it moves faster than the value you found in part 'd', $v(t = 0) > v_c$. Will the rod slow down? Why?

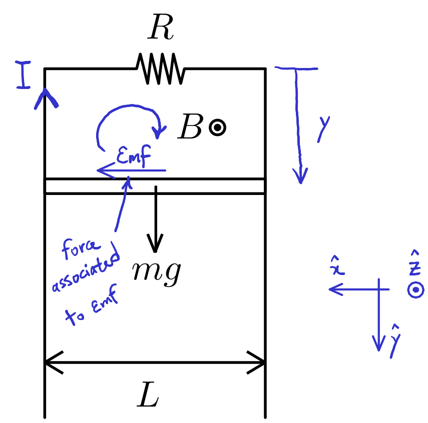


Figure 1: A rod sliding down in a magnetic field.

a) $\mathcal{E} = -vBL$ (motional emf), direction of the force associated to this emf is shown in the figure

or

$$\Phi_B = \cancel{N} BA \cancel{\cos\phi} = BLy$$

$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t} = -BL\frac{\Delta y}{\Delta t} = -BLv$$

clockwise, as ⊙ and ↻ defined as positive direction

or Lenz's law: flux is increasing in + direction, i.e. ⊙, the induced current should give us magnetic field in the opposite direction, i.e. ⊗, which using right-hand rule, we realize the induced current is clockwise.

motional emf in a sense is equivalent to Faraday's law.

exact solution for e & f:

downward acceleration

$$m\frac{dv}{dt} = mg - \frac{B^2L^2}{R}v$$

velocity at time t

$$\Rightarrow \int_{v(0)}^{v(t)} \frac{dv}{\underbrace{\boxed{\frac{mgR}{B^2L^2}}}_{v_c} - v} = \underbrace{\boxed{\frac{B^2L^2}{mR}}}_{g/v_c}\int_0^t dt$$

initial velocity

$$\Rightarrow v(t) = v(0)e^{-gt/v_c} + v_c\left[1 - e^{-gt/v_c}\right].$$

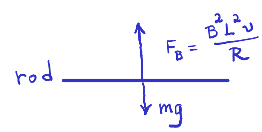
b) $I = \dfrac{\mathcal{E}}{R} = -\dfrac{BLv}{R}$, the direction as discussed is clockwise.

c) on current I, magnetic force is, $\vec{F} = \vec{I}\ell \times \vec{B}$ which gives us,

$$\vec{F}_B = \left[|I|\, L\, \hat{x}\right] \times B\hat{z} = \frac{BLv}{R}LB\,(\hat{x}\times\hat{z}) = \frac{B^2L^2v}{R}(-\hat{y})$$

upward

d) $\sum F_y = ma_y = 0 \longrightarrow mg - \dfrac{B^2L^2v_c}{R} = 0 \Rightarrow \boxed{v_c = \dfrac{mgR}{B^2L^2}}$



if $v = v_c$ $F_B = mg$

e) $a = g - \dfrac{B^2L^2}{mR}v$

initially $v = 0 \rightarrow F_B = 0$ so it accelerates $a = g$

the speed increases $\rightarrow$ acceleration decreases, in fact $v \rightarrow v_c$ and $a \rightarrow 0$ as $t \rightarrow \infty$.

f) initially $v > v_c \rightarrow F_B > mg \rightarrow$ so it accelerates upward or it slows down till it reaches $F_B = mg$ or $v = v_c$.

$v \rightarrow v_c$, $a \rightarrow 0$ as $t \rightarrow \infty$.