

Name:

You can use any two lines
(not parallel) as coordinate basis.

1) Linear Momentum

A fireworks rocket is moving at a speed of 45.0 m/s. The rocket suddenly breaks into two pieces of equal mass, which fly off with velocities \mathbf{v}_1 and \mathbf{v}_2 , as shown in the fig. 1. What are the magnitudes of the velocities \mathbf{v}_1 and \mathbf{v}_2 ? [1.0 pt]

This is a verbatim copy of the problem 62, Chapter 7, Physics, by Cutnell & Johnson, 9th edition.

There is no external force acting on the rocket during the time it breaks into pieces. So momentum of the rocket is conserved,

$$\vec{F}_{\text{ext}} = 0 \rightarrow \vec{P} = \text{const. or } \vec{P}_f = \vec{P}_i$$

with the choice of coordinates drawn,

$$P_{ix} = P_{fx} \rightarrow m \cdot 45 \frac{m}{s} = \frac{m}{2} v_1 \cos 30^\circ + \frac{m}{2} v_2 \cos 60^\circ \quad (i)$$

$$P_{iy} = P_{fy} \rightarrow 0 = \frac{m}{2} v_1 \sin 30^\circ - \frac{m}{2} v_2 \sin 60^\circ \quad (ii)$$

$$(ii) \rightarrow v_1 = v_2 \sqrt{3} \quad (i) \rightarrow \frac{3}{4} v_2 + \frac{1}{4} v_2 = 45 \frac{m}{s}$$

$$\rightarrow v_2 = 45 \frac{m}{s} \quad \& \quad v_1 = 78 \frac{m}{s}$$

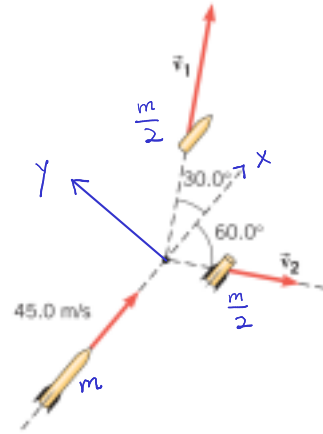


Figure 1: A rocket breaking to two equal mass parts.

2) Rotational Kinematics

You are riding a bike. At each time $T = 5$ s you are doing a total turn with pedals. The gear in front has radius $r_F = 15$ cm and the gear at the back has radius $r_B = 5$ cm. The radius of the wheels are $R = 30$ cm. There is no sliding involved. Find the velocity of the bike. [1.0 pt]

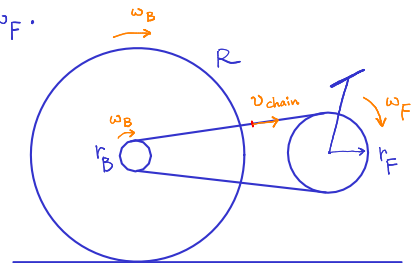
The angular velocity of the front gear and the pedals is the same, ω_F .

$$T = 5 \text{ s} \rightarrow \omega_F = \frac{2\pi}{T} = \frac{2\pi}{5} \frac{\text{rad}}{\text{s}}$$

$$v_{\text{chain}} = r_F \omega_F = r_B \omega_B \rightarrow \omega_B = \frac{r_F}{r_B} \omega_F = \frac{6\pi}{5} \frac{\text{rad}}{\text{s}}$$

The angular velocity of the back gear and the wheel

is the same, ω_B . So, $v_{\text{bike}} = R \omega_B = 36 \pi \frac{\text{cm}}{\text{s}} = 1.13 \frac{\text{m}}{\text{s}}$.



3) Rotational Dynamics

A person with mass $m = 70$ kg is doing push-ups. Holding the position as shown in fig. 2, find the normal force exerted by the floor on each hand and each foot. $l_1 = 0.41$ m and $l_2 = 0.84$ m. [1.0 pt]

This is a copy of the problem 12, Chapter 9, Physics, by Cutnell & Johnson, 9th edition.

The person is at equilibrium

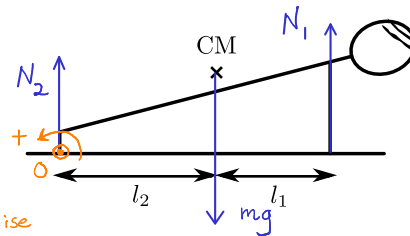
no acceleration: $\left\{ \begin{array}{l} \sum \vec{F} = 0 \text{ (two eq'n in 2D)} \end{array} \right.$

no angular acceleration: $\left\{ \begin{array}{l} \sum \tau_o = 0 \text{ (one eq'n in 2D, you can choose any point O on the plane.)} \end{array} \right.$

$$\sum F_x = 0 \rightarrow \text{trivial.}$$

$$\sum F_y = 0 \rightarrow N_1 + N_2 - mg = 0 \quad (i)$$

$$\sum \tau_o = 0 \rightarrow N_2 \cdot 0 - mg l_2 + N_1 (l_1 + l_2) = 0 \quad (ii)$$



Counter clockwise
direction is the
positive direction

Figure 2: A person doing push-up.

$$(ii) \rightarrow N_1 = mg \frac{l_2}{l_1 + l_2} = 461 \text{ N.}$$

$$(i) \rightarrow N_2 = mg \frac{l_1}{l_1 + l_2} = 225 \text{ N.}$$

$$F_{\text{hand}} = \frac{N_1}{2}, \quad F_{\text{foot}} = \frac{N_2}{2}$$