

For full credit, be sure to show all your work.

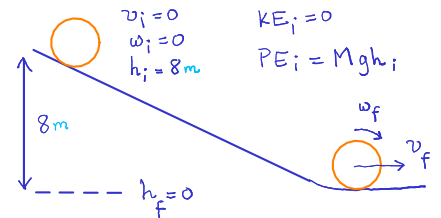
1. A solid sphere (moment of inertia $I = \frac{2MR^2}{5}$) with mass 0.250 kg and radius 0.100 m starts from rest on top of a 8.00 m hill and rolls down to the bottom. Find the magnitude of the translational velocity when the sphere reaches the bottom of the hill. (1 point)

We know the sphere is rolling without sliding $\rightarrow v_f = R\omega_f$ (i)

$$W_{n.f.} = E_f - E_i \rightarrow E_i = E_f \rightarrow KE_i + PE_i = KE_f + PE_f$$

$$\rightarrow 0 + Mgh_i = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 + 0 \xrightarrow{(i)} Mgh_i = \frac{1}{2} M v_f^2 + \frac{1}{2} I \left(\frac{v_f}{R}\right)^2$$

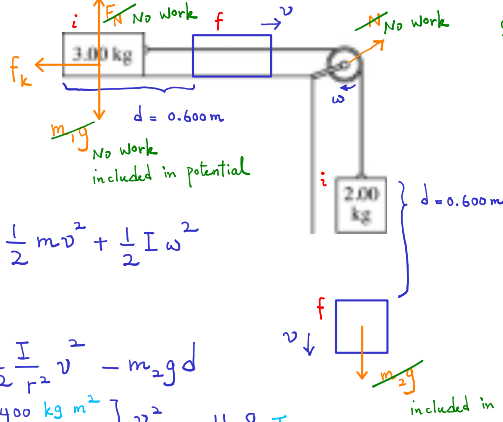
$$\rightarrow v_f = \sqrt{\frac{2gh_i}{(1 + \frac{I}{MR^2})}} = \sqrt{\frac{10}{7}gh_i} = 10.6 \frac{m}{s} = \frac{1}{2} \left(M + \frac{I}{R^2}\right) v_f^2$$



2. In the figure, two blocks, of masses 2.00 kg and 3.00 kg, are connected by a light string that passes over a frictionless pulley of moment of inertia 0.00400 kg · m² and radius 5.00 cm. The coefficient of friction for the tabletop is 0.300. The blocks are released from rest. Using energy methods, find the speed of the upper block just as it has moved 0.600 m. (1 point)

$$W_{n.f.} = E_f - E_i$$

$$W_{f_k} = (\mu_k m_1 g) d \cos(180^\circ) = -\mu_k m_1 g d$$



The tension force is an internal conservative force. So the net work done by the tension force on the system of masses & pulley is zero.

No sliding, i.e. the string does not slide on the pulley $\rightarrow v = r\omega$ (i)

$$\begin{cases} KE_i = 0 \\ PE_i = 0 \end{cases} \begin{cases} KE_f = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} I \omega^2 \\ PE_f = -m_2 g d \end{cases}$$

$$\xrightarrow{(i)} -\mu_k m_1 g d = \frac{1}{2} (m_1 + m_2) v^2 + \frac{1}{2} \frac{I}{r^2} v^2 - m_2 g d$$

$$-5.3 \text{ J} = \frac{1}{2} \left[5.00 \text{ kg} + \frac{0.00400 \text{ kg m}^2}{(5.00 \text{ cm})^2} \right] v^2 - 11.8 \text{ J} \rightarrow v = 1.4 \frac{m}{s}$$

3. A 5.00 kg mass is attached to a spring with spring constant 75.0 N/m. If the spring oscillates with simple harmonic motion, (a) find the period T and (b) find the mass that would be required to double the period to 2T. (1 point)

(a) $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{5.00}{75.0}} \text{ sec} = 1.62 \text{ sec}$

(b) $\tilde{T} = 2T = 2\pi \sqrt{\frac{\tilde{m}}{k}} \rightarrow \tilde{m} = \frac{\tilde{T}^2}{4\pi^2} k = \frac{(2T)^2}{4\pi^2} k = 4m = 20.0 \text{ kg}$