General Physics 204, Section ... Monday, Feb 23, 2015

Quiz #4

Name:

$\hat{x}$: unit vector showing $+x$ direction
$\hat{y}$: unit vector showing $+y$ direction
$\hat{z}$: unit vector showing $+z$ direction

right-handed Cartesian coordinates

$\hat{x} \times \hat{y} = \hat{z}$
$\hat{y} \times \hat{z} = \hat{x}$
$\hat{z} \times \hat{x} = \hat{y}$

1) CLASS(2)

Hans Christian Oersted fixes four parallel wires perpendicular to the plane (parallel to z-axis), in a square pattern with side length $a = 20$ cm. The current passing through these wires are $I = 1.0$ A with directions shown in the figure. An electron is at point A and moving with velocity $\mathbf{v} = -\hat{\mathbf{x}}2.0 \times 10^6$ m/s. Find the force implied to this electron. **[4 pts]**

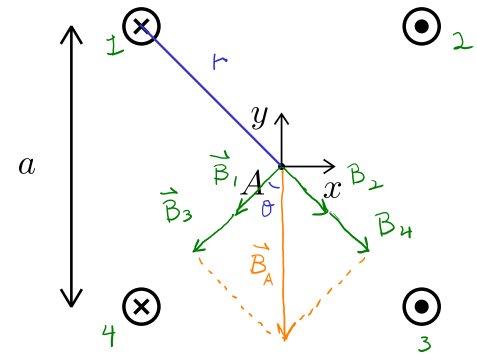


Figure 1: Parallel wires.

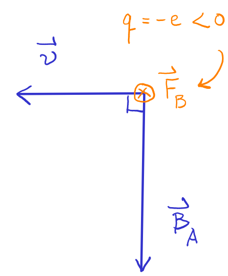
First we find $\vec{B}_A$, the magnetic field at A,

$$\vec{B}_A = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \underbrace{(-\hat{y})}_{\text{direction}}\, 4 B_1 \cos\frac{\pi}{4}$$

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi a}\sqrt{2}$$

$$\rightarrow \vec{B}_A = (-\hat{y})\frac{4\mu_0 I}{2\pi a}\sqrt{2}\cos\frac{\pi}{4} = (-\hat{y})\frac{4\mu_0 I}{2\pi a} = (-\hat{y})\, 4\ \mu T.$$

Now electron interacts w/ field, $\vec{F}_B = q\vec{v} \times \vec{B}_A$, $q = -e$

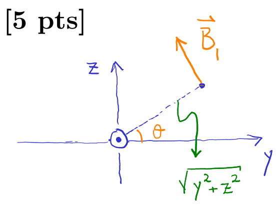


$$\left.\begin{array}{l}\vec{v} = (-\hat{x})\, v \\ \vec{B}_A = (-\hat{y})\, B_A\end{array}\right\} \vec{v} \times \vec{B} = (-\hat{x}) \times (-\hat{y})\, v B_A = \hat{x} \times \hat{y}\ v B_A = \hat{z}\, v B_A.$$

$$\rightarrow \vec{F}_B = (-\hat{z})\, e v B_A = (-\hat{z})\, 1.28 \times 10^{-18}\ N.$$

2) CLASS(2)

William Gilbert is asking you to imagine the Cartesian coordinate system, (x, y, z), and consider there is a current I passing through each axis. What is the magnetic field at any point $B(x, y, z)$? **[5 pts]**



→ Contribution of current I on x-axis: $\vec{B}_1 = \frac{\mu_0 I}{2\pi\sqrt{y^2+z^2}}\left(-\sin\theta\,\hat{y} + \cos\theta\,\hat{z}\right)$

$$= \frac{\mu_0 I}{2\pi}\frac{1}{y^2+z^2}\left(-z\hat{y} + y\hat{z}\right),$$

→ The same calculation holds for $\vec{B}_2$, contribution of current I on y-axis, if we snap $x \to y \to z \to x$,

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi}\frac{1}{z^2+x^2}\left(-x\hat{z} + z\hat{x}\right),$$

→ Similarly for $\vec{B}_3$, contribution of current I on z-axis, $\vec{B}_3 = \frac{\mu_0 I}{2\pi}\frac{1}{x^2+y^2}\left(-y\hat{x} + x\hat{y}\right)$.

$$\vec{B}(x, y, z) = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{2\pi}\left(\frac{-z\hat{y}+y\hat{z}}{y^2+z^2} + \frac{-x\hat{z}+z\hat{x}}{z^2+x^2} + \frac{-y\hat{x}+x\hat{y}}{x^2+y^2}\right)$$

3) CLASS(2)

John Michell shape a piece of wire as a triangle, RST, and connects it to a current source. The current passing through the triangle is $I = 2.0$ A. The lengths are $|RS| = 6$ cm, $|ST| = 10$ cm, and $|TR| = 8$ cm. He turns on a uniform magnetic field as shown in fig. 2, $\mathbf{B} = 0.2$ T$\hat{\mathbf{x}}$. Find the torque vector (the direction of the torque will show the axis and the direction of rotation). **[3 pts]**

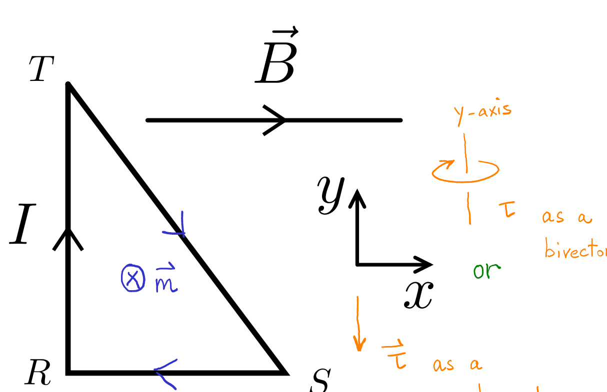


Figure 2: Parallel wires.

$$A = \tfrac{1}{2}(6\text{ cm})(8\text{ cm}) = 2.4 \times 10^{-3}\ m^2.$$

$$\vec{m} = I\vec{A} = (-\hat{z})\, 4.8 \times 10^{-3}\ Am^2.$$

$$\vec{\tau} = \vec{m} \times \vec{B} = (-\hat{z}) \times (\hat{x})\ 9.6 \times 10^{-4}\ Nm = (-\hat{y})\ 9.6 \times 10^{-4}\ Nm.$$