

Name:

Earth's frame:

Spaceship's frame:

event 1.  $(t_1^E=0, x_1^E=0)$

event 1.  $(t_1^S=0, x_1^S=0)$

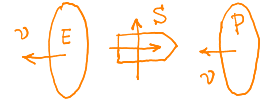
1) CLASS(2)

event 2.  $(t_2^E, x_2^E=20 \text{ ly})$

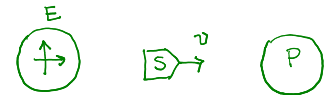
event 2.  $(t_2^S, x_2^S=0)$   $t_2^S$  : proper time

Hendrick Lorentz is 52 years old and starts to travel to another close planet which is 20 ly away. The spaceship velocity is  $9c/10$ . How old is he when he arrives to the other planet? [4 pts]  $1 \text{ ly} = 1 \text{ cy}$

Spaceship frame of ref. :  $L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = 20 \text{ ly} \sqrt{1 - \frac{9^2}{10^2}} = 2\sqrt{19} \text{ ly}$   
 $\rightarrow \Delta t_0 = \frac{2\sqrt{19} \text{ ly}}{\frac{9}{10} c} = \frac{20\sqrt{19}}{9} \text{ y} = t_2^S$



Earth/planet frame of ref. :  $L_0 = 20 \text{ ly} \rightarrow \Delta t = \frac{20 \text{ ly}}{\frac{9}{10} c} = \frac{200}{9} \text{ y} = t_2^E$   
 $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow \Delta t_0 = \frac{20}{9} \sqrt{19} \text{ y} = t_2^S$



H. L.'s age =  $52 + \frac{20\sqrt{19}}{9} \text{ y}$

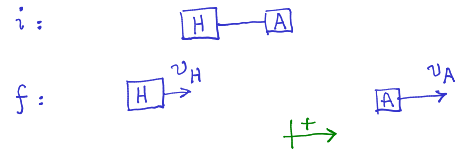
2) CLASS(2)

Henri Poincaré and Albert Einstein are on top of the ice. Take their rest masses to be  $m_H = 3/2 m_A$ . The surface of the ice is frictionless and the speed of light is  $c = 3 \text{ m/s}$ . As they were standing at rest, they push each other away. We know  $v_A = 9c/10$ . Find  $v_H$ . [6 pts]

There is no external force in horizontal direction

$$F_{\text{ext}} = \frac{\Delta P}{\Delta t} = 0$$

$$\rightarrow P_i = P_f \quad (\text{momentum is conserved}).$$



$$\begin{cases} P_i = 0 \\ P_f = \frac{m_A v_A}{\sqrt{1 - \frac{v_A^2}{c^2}}} + \frac{m_H v_H}{\sqrt{1 - \frac{v_H^2}{c^2}}} \end{cases} \rightarrow \frac{m_A v_A}{\sqrt{1 - \frac{v_A^2}{c^2}}} + \frac{m_H v_H}{\sqrt{1 - \frac{v_H^2}{c^2}}} = 0$$

$$\rightarrow \frac{m_A^2 v_A^2}{1 - \frac{v_A^2}{c^2}} (1 - \frac{v_H^2}{c^2}) = m_H^2 v_H^2 \rightarrow v_H = -c \left[ 1 + \frac{m_H^2}{m_A^2} \left( \frac{c^2}{v_A^2} - 1 \right) \right]^{-1/2}$$

$$= -3 \left[ 1 + \frac{9}{4} \cdot \frac{19}{81} \right]^{-1/2} = -\frac{18}{\sqrt{55}} \text{ m/s}$$

Kinetic energy is clearly not conserved:

$$KE_i = 0$$

$$KE_f = m_A c^2 \left( \frac{1}{\sqrt{1 - \frac{v_A^2}{c^2}}} - 1 \right) + m_H c^2 \left( \frac{1}{\sqrt{1 - \frac{v_H^2}{c^2}}} - 1 \right) > 0.$$

$$KE_f = \text{Interaction Energy} \neq 0.$$