

Name:

1) Pieter van Musschenbroek connects his capacitor with capacitance $C = 2.2 \text{ mF}$ to a battery with emf $\mathcal{E} = 4.0 \text{ V}$ and a resistor $R = 20 \text{ k}\Omega$ in series. The capacitor was free of charge before connecting the circuit, at time $t = 0$.

a) Calculate the time constant, τ . [2 pts]

b) How long it takes for the capacitor to get charged with $Q = 4.4 \text{ mC}$?

Remember that $V_C(t) = \mathcal{E}(1 - e^{-t/\tau})$ and $Q = CV$. How much is the current at this time? [3 pts]

c) Pieter measures the resistor voltage through time and fits a curve $y = 3.89e^{-0.023x} + 0.03$. Write down the units of these numerical fit parameters and compare them to the theoretical values. [4 pts]

a) $\tau = RC = 20 \text{ k}\Omega \cdot 2.2 \text{ mF} = 44 \text{ s}$.

b) We call the time t_1 . We know,

$$Q(t=t_1) = 4.4 \text{ mC} = CV_C(t=t_1) = 2.2 \text{ mF} \cdot \mathcal{E}(1 - e^{-t_1/\tau}) = 8.8 (1 - e^{-t_1/\tau}) \text{ mC}.$$

$$\text{So } (1 - e^{-t_1/\tau}) = 0.50 \rightarrow t_1 = \tau \ln 2 = 30 \text{ s}.$$

$$V_C(t=t_1) = 2.0 \text{ V} \rightarrow V_R(t=t_1) = \mathcal{E} - V_C(t=t_1) = 2.0 \text{ V} \rightarrow I(t=t_1) = \frac{V_R(t=t_1)}{R} = 0.10 \text{ mA}.$$

c) Theoretically $V_R(t) = \frac{\mathcal{E}}{R} e^{-t/\tau} + 0 \rightarrow y = (3.89 \text{ V}) e^{-(0.023 \frac{1}{\text{s}})x} + 0.03 \text{ V}$. $\frac{1}{\tau} = 0.023 \frac{1}{\text{s}}$.

2) Hertha Ayrton uses the same setup that we had in 'RLC Circuits' lab, part B, (R and L and C in series connected to a battery) and measures the current I (or voltage V_R) vs time as shown in fig. 1. She knows $R = 100\Omega$ and wants to find L and C .

a) Roughly measure the period of oscillations, T . Calculate f and ω . [3 pts]

b) Roughly measure the time constant, τ . Remember that the peaks are roughly decaying like $e^{-t/\tau}$. [3 pts]

c) Find L and C . [2 pts]

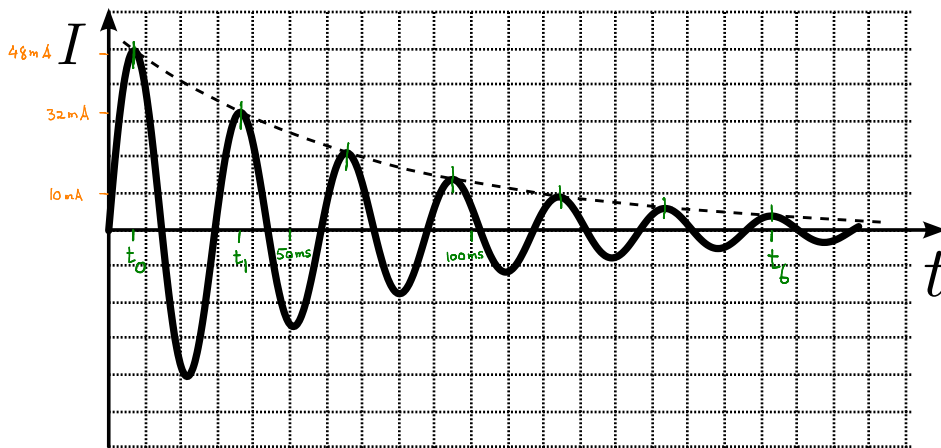


Figure 1: A series RLC circuit's current vs time. Each square in the grid is 10 mA by 10 ms.

a) $t_6 = t_0 + 6T$. $t_6 = 183 \pm 1 \text{ ms}$, $t_1 = 7 \pm 1 \text{ ms} \rightarrow T = 29.3 \pm 0.2 \text{ ms}$.

$$\omega = \frac{2\pi}{T} = 214 \frac{\text{rad}}{\text{s}}, f = \frac{1}{T} = 34.1 \text{ Hz}.$$

b) $t_1 - t_0 = T = 29.3 \text{ ms}$. $\frac{I(t_1)}{I(t_0)} \approx e^{-T/\tau} \rightarrow \tau = \frac{T}{\ln(\frac{48}{32})} = 72 \text{ ms}$.

c) $\tau = \frac{2L}{R} = 72 \text{ ms} \rightarrow L = \frac{\tau R}{2} = 3.6 \text{ H}$.

$$\omega \approx \frac{1}{\sqrt{LC}} \rightarrow LC = 2.18 \times 10^{-5} \text{ s}^2 \rightarrow C = 6.1 \text{ }\mu\text{F}.$$