**1)** Consider a sphere with radius $R$ with a hollow inside with radius $r$. The surface of this hallow touches the sphere, as shown in fig. 1. The density of the this sphere is $\rho$. There is a mass $m$ which is at a distance $d$ from the center of the sphere. The center of this mass, the center of hollow, and the center of the sphere are collinear.
**a)** Find the gravitational force between the sphere and the mass.
**b)** Find the gravitational energy of this system.

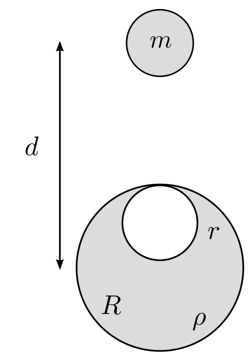

Figure 1: The gravitational force between spheres.

**2)** There is a uniform rod with a mass $M$ and length $L$, with two ends sitting on the origin and $(-L, 0, 0)$, and a mass $m$ sitting on the $(x, 0, 0)$.
**a)** Find the gravitational force between the rod and the mass.
**b)** Find the gravitational potential energy of the system.

**3)** A planet with a uniform density $\rho$ will have a gravitational field inside it, given by $\mathbf{g}(\mathbf{r}) = -4\pi G\mathbf{r}/3$, where $\mathbf{r}$ is the vector from center of the planet. Consider such a planet with radius $R$ and center at the origin, i.e. $(0, 0, 0)$.
**a)** Find the pressure at a radius $r$.
**b)** If there is a spherical hollow inside the sphere, with radius $b$, centered at $\mathbf{d} = (0, 0, d)$, find the gravitational field inside this hollow. What are the equipotential surfaces inside this hollow?

**4)** A mass $m$ is sitting at a distance $x$ from the center of a ring with a mass $M$ and radius $R$. The mass $m$ is on the axis of this ring.
**a)** Find the gravitational force acting on $m$.
**b)** If you release the mass $m$, say from $x = d$, what will be its velocity when passing through the ring, i.e. when $x = 0$.

**5)** By observing the movement of the distant stars, we can infer that there are planets orbiting it. We can measure the line of sight velocity, the velocity that the star moves away from and towards us. By observing the star 14 Herculis we measured the graph as shown in fig. 2. With other measurements the mass of the star is estimated to be 0.9 times the mass of the sun. Assume that there is only one planet orbiting this star, and also our view is along the planet's orbit plane.
**a)** Estimate the mass of the planet.
**b)** Estimate the orbital radius of the planet.

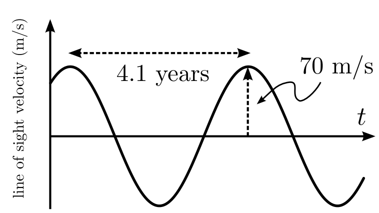

Figure 2: The line of sight velocity of star 14 Herculis.

**6)** A satellite is too close to the earth, so it is losing its mechanical energy, $E$, by the air drag force, $\mathbf{f} = -bv\hat{\mathbf{v}}$. This satellite is in a circular orbit of radius $R$ to begin with, and the process of loosing energy is really slow; so you can think about the motion as a circular motion, with a radius which shrinks slowly.
**a)** Calculate the energy loss during each revolution and call it $\Delta E$.
**b)** Using the approximation, $\mathrm{d}E/\mathrm{d}t \approx \Delta E/T$, where $T$ is the period of a revolution, estimate $r(t)$, the radius as a function of time.