Physics 271, Sections H1 & H2 — Thursday, Oct 16, 2014

Problems #6 — Potential Energy & Energy Conservation

1) A mass m is released from point A with height H in a frictionless path as shown in fig. 1. The mass leaves the circular loop at point D before reaching the top, at angle θ. The numerical values are, $\sin\theta = 4/5$, and $R = 10$ cm, where R is the radius of circular loop.

a) Write the equation of motion in radial direction at point D, and find the velocity at D.
b) Show that the height at point D is $h = R(1+\cos\theta)$ and calculate h.
c) Using energy conservation find H.
d) Now if there is friction from B to C, with coefficient $\mu_k = 0.2$ and $l = 10$ cm, where l is the distance between B and C, how much we need to increase H so the mass will leave the circular path at the same point D.

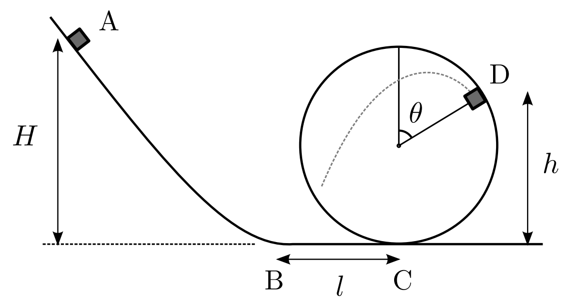

Figure 1: The path.

2) There is a vertical spring attached to the ground, with spring constant k, as shown in fig. 2. Call vertical direction y, the direction up is positive, and $y = 0$ is where initially the plate on top of the spring is. This origin, $y = 0$ is not moving with respect to the ground; the reason we put it there is, not to care about the length of the spring and make the notation simpler. We release a box with mass m from a height H. Answer in terms of H, g, k, and m.
a) What is the velocity of box when reaches $y = 0$?
b) Find the minimum height, $y_{\min}$, that the box will reach.
c) Draw a potential energy graph for the system and discuss the motion of the box on it.
d) Find y_0, the point that the box has the maximum speed. Discuss the normal force between the box and the plate on top of the spring.

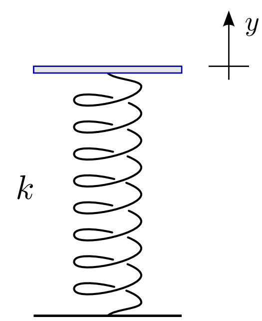

Figure 2: A vertical spring.

3) There is a horizontal spring, with spring constant k, is attached to a wall, as shown in fig. 3. There is a box with mass m, initially at $x = x_0$, moving with velocity v_0 towards the spring.

a) If there is no friction how much the spring will be compressed?
b) If the coefficient of kinetic friction between the box and the ground is μ_k, how much the spring will be compressed?
c) Solve above parts 'a' and 'b', if instead of horizontal ground, the mass and spring are on a inclined plane with an angle θ.

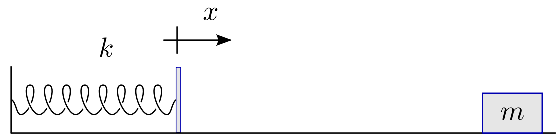

Figure 3: A horizontal spring.

4) You are sitting on top of a spherical ice surface with radius R. There is no friction between you and ice. You start sliding down.
a) Find the velocity when you reach height h measured from the center of the sphere.
b) Find the normal force as a function of h.
c) Where will you detach from the sphere and start a projectile motion?

5) Consider a horizontal mass and spring system as shown in fig. 4. The mass has friction with the ground. Take both the coefficients of kinetic and static friction to be $\mu_s = \mu_k = \mu$. We pull the mass to the position $x = X_0$ and release. We call the consecutive furthest points from $x = 0$ that the mass stops at, $X_1, X_2, \ldots$, i.e. the mass starts moving from the rightmost X_0 to leftmost X_1 where it stops, then it comes back to rightmost point X_2 where it stops again and so on. You see, because of the friction $X_0 > X_1 > X_2 > \ldots$.

a) Write down an equation for conservation of energy, moving from $x = X_i$ to $x = X_{i+1}$.
b) Simplify the above equation and discuss the situations $X_i X_{i+1} > 0$ and $X_i X_{i+1} < 0$ and $X_i X_{i+1} = 0$.
c) How many times the mass will pass through $x = 0$?

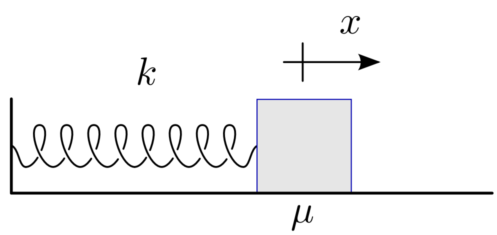

Figure 4: A horizontal mass spring system.