Physics 271, Sections H1 & H2 — Thursday, Oct 23, 2014
Problems #7 — Linear Momentum

1) center of mass
Find the center of mass for each one of the configurations shown in fig. 1. The bottom right is a tetrahedron. And the other figures are planar with a uniform mass distribution. Where is the center of mass for a triangular-shaped frame?

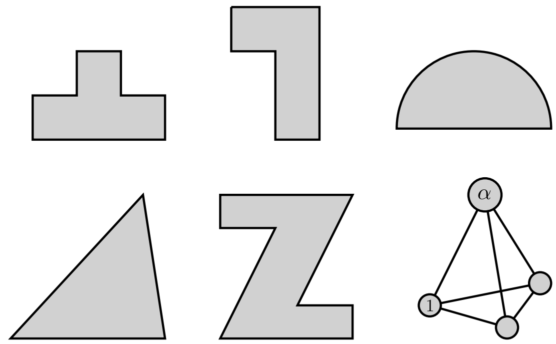

Figure 1: Six mass distributions.

2) Consider two masses m in one dimension. These masses are moving with initial velocities v_{1i} and v_{2i}.
a) Show that if an elastic collision happens, then for final velocities we have $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$.
Above result has an interesting interpretation. If the collision happens really fast (hard collision), you can think about these masses passing through each other without any collisions.
b) Using the above result, explain different scenarios in Newton's cradle. For example, say we have five balls in total; and we pull away three of them and let them fall. How will the system behave?

3) We shot a bullet with mass m and initial velocity of v_0 to a box with mass M sitting on the ground. See fig. 2. There is no friction between the box and the ground.

a) If the bullet gets stuck with the box, find the velocity of the box after the collision. How much heat is produced in this totally inelastic collision?
b) If the bullet pierces through the box, and continue moving with $v_0/2$, again, find the velocity of the box after the collision and the amount of heat which is produced in this collision.

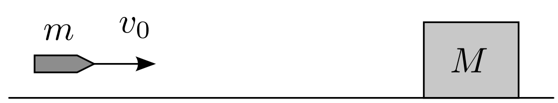

Figure 2: A bullet colliding to a box.

4) Consider two masses m connected with a spring with spring constant of k and relaxed length of l. You are holding this system from the top mass. See fig. 3. You let the system go at time $t = 0$.
a) Find $y_{\text{CM}}(t)$, the height of the center of mass as a function of time.
b) Explain the motion of the masses.

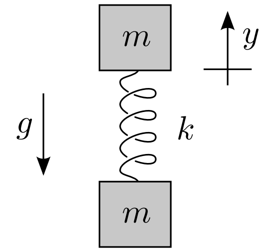

Figure 3: Two masses connected with a spring falling.

5) You plus $N-1$ of your friends are standing on the edge of a flat wagon. Each of you has a mass m and the wagon has a mass M. The length of the wagon is L.
a) If n of you move to the other end of the wagon, how much will the wagon move?
b) How fast the wagon will go if all of you, together, jump out with velocity u relative to the wagon?
c) How fast the wagon will go if you jump one-by-one with velocity u relative to the wagon?

6) Consider these two somewhat similar problems. In both cases, a small mass m is moving initially with velocity v_0, towards a mass M. The mass M is initially at rest. The is no friction and the mass M can move freely on the ground in both cases.

a) In the first case, there is a spring attached to the mass M. Explain the motion and find the maximum compression of the spring.
b) In the second case, the mass M has an inclined plane on the side which mass m can climb without any friction.Again, explain the motion, and this time find the maximum height that the mass m reaches.

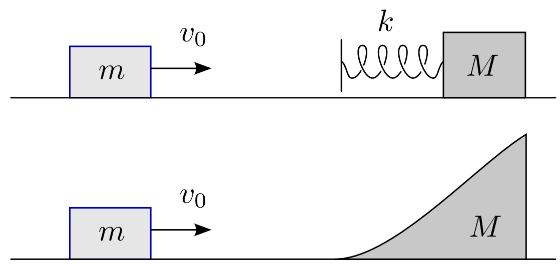

Figure 4: The maximum compression and height.

7) Consider two disks with radius r and mass m, on a two dimensional plane. We sit on a frame of reference that the mass number two is at rest initially (this frame usually referred to as the lab frame), and mass number one is moving towards it with velocity $\mathbf{v}_0 = v_0\hat{\mathbf{x}}$. These masses collide elastically with parameter of collision b. See fig. 5.

a) Write down all the conservation equations. As you see we have three equations but four unknowns.
b) Show that the disks' velocities are perpendicular after the collision (in the lab frame).
c) Discuss the parameter of collision and find another equation. Solve for $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$.
d) Discuss the problem and solve it again, this time in the center of mass frame of reference (the CM frame).

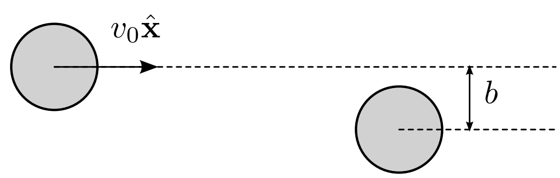

Figure 5: Two disks colliding with parameter of collision b.

8) Consider a mass m between the wall and a heavier mass M, moving with velocity v_0. The mass M is initially at rest.

a) Assume all the collisions are elastic. Write down (v_{n+1}, V_{n+1}), the positive velocities of m and M after n-th collision of the masses (and n collisions of m to the wall.), in terms of (v_n, V_n).
b) Solve above equations to find (v_n, V_n) in terms of n. These equations are mostly referred to as difference equations, and solving them is equivalent to finding the n-th power of a two by two matrix.
c) If $M = 9m$, how many times will the masses collide?
d) Solve above parts when the coefficient of restitution for the collision between the masses is equal to η. Take the wall collisions to be elastic.

Figure 6: The bouncing ends.