1) rotational kinematics
You are riding a bike. At each time T you are doing a total turn with pedals. The gear in front has radius r_{F} and the gear at the back has radius r_{B}. The radius of the wheels are R. See fig. 1.
a) Find the angular velocity of the front gear.
b) Find the angular velocity of the back gear.
c) Find the velocity of the bike.
d) If you speed up the pedal turning by angular acceleration of α, what would be the bike's acceleration?
e) [dynamics] Your mass is M and bike's mass is m. You are initially at rest. You start pedaling with constant torque τ. Neglecting the rotational kinetic energy terms, how fast will you go after n full turns of pedals.

Figure 1: A bike.

2) moment of inertia
a) [quiz related] Find the moments of inertia, I_x, I_y, I_z, for the letter E, consisting of four uniform rods with mass m and length l, welded together. See fig. 2.
b) [perpendicular axis theorem] Consider a two dimensional sheet at the xy plane. Prove that $I_z = I_x + I_y$, where I_α is the moment of inertia with respect to the axis α. Find the moment of inertia for a uniform solid plane circular disk with mass m and radius r with respect to its diagonal. What if the axis is a tangent line to the disk?
c) Consider a uniform solid plane triangle with mass m. Find its moment of inertia with respect to one of its sides.
d) [for fun] Find the moment of inertia of a uniform solid plane triangle with respect to the axis perpendicular to its plane, passing through one of its vertices. Now, find the moment of inertia for a plane N-sided polygon with respect to an axis perpendicular to its plane. Use notation of your choice to represent the triangle or the polygon.
e) [for fun] Find the moment of inertia of a N-sided polygon if it is only the frame, not solid. Take mass per unit length to be 1.

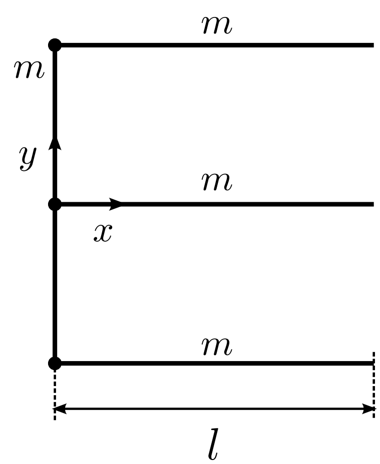

Figure 2: A letter E.

3) Consider a mass m_1 hanging and connected to another mass m_2 with a massive pulley with mass M, moment of inertia I, outer radius R, and inner radius r. The string does not slide on the pulley. The mass m_2 is on a horizontal plane and moves without friction. See fig. 3.
a) Draw a free body diagram for m_1, m_2, and the pulley.
b) Write down the equations of motion for the masses and the pulley.
c) Write down equations for the condition that the string does not slide on the pulley.
d) Find the acceleration and the tension forces.

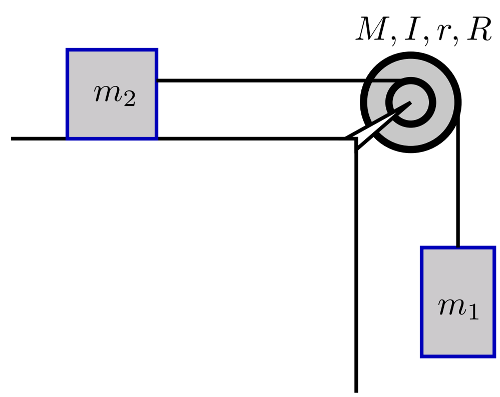

Figure 3: A hanging mass connected to another mass with a massive pulley.

4) A uniform ruler with mass m and length L is sitting at the edge of the table. We hold the ruler in a way that only the length $l < L/2$ of it stays on the table, and then we release the ruler. You can think of the table edge as the fulcrum.
a) Find the initial accelerations of both ends of this ruler.
b) Find the initial angular acceleration of the ruler.
c) Find the initial normal force on the ruler from the fulcrum.

5) Consider a thin uniform rod, with mass m and length L, which can pivot about a horizontal frictionless pin. See fig. 4. The pin is positioned at a distance l from the center of the rod. We release this rod from an angle θ. Write down the energy conservation and find the angular velocity of the rod when it is horizontal.
[quiz related]

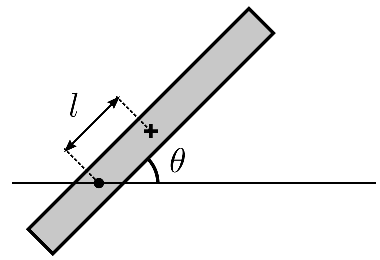

Figure 4: A rod rotating around a pivot.

6) A yo-yo is falling down as shown in the fig. 5. This yo-yo has an inner radius r, mass M, and the moment of inertia about the axis perpendicular to circles, passing through the center of it, is I.

a) Find a relation between the acceleration of the center of the yo-yo and the angular acceleration of the yo-yo.
b) Draw the free body diagram and write down the equations of motion for this yo-yo.
c) Find the tension force and acceleration.

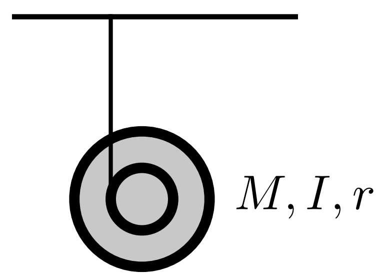

Figure 5: A yo-yo falling down.