1) Two uniform disks with radii R_1 and R_2, masses m_1 and m_2, and moments of inertia I_1 and I_2, are rotating with angular velocities ω_1 and ω_2. We brought these disks together and they touch at a point. There is friction between the disks so disks reach final angular velocities Ω_1 and Ω_2 which we are trying to calculate. See fig. 1.
a) Write a condition for Ω_1 and Ω_2, knowing than the disks are not sliding relative to each other.
b) What quantity is conserved here? Find the angular velocities Ω_1 and Ω_2.
c) If there is a friction force f between the disks while they were sliding, how long it took for the disks to reach the final velocities?

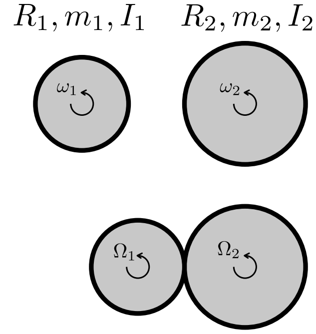

Figure 1: Two disks.

2) A uniform spherical ball with radius r and mass m is rolling down a slope from height H and enters a vertical circular path with radius r, as shown in fig. 2.

a) Find the condition on H, so that the ball stays on the circular path. Assume that the ball is rolling the whole time.
b) Find the condition on H, so that the ball detaches from the path at height h. Assume the ball is rolling the whole time.
c) As the ball is slowing down on the circular path the static friction must slow down the rotation so that the ball rolls the whole time. Discuss the condition on the coefficient of static friction, μ_s, so that the ball rolls the whole time. Explain why in part 'b' the assumption is not valid, if μ_s is finite.

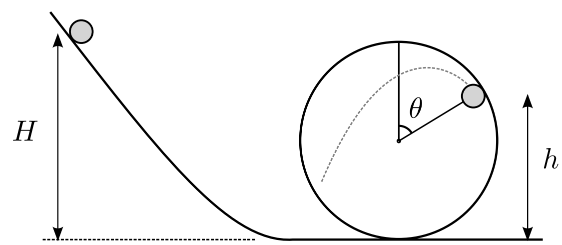

Figure 2: A sphere rolling.

3) You are hitting a ball (uniform sphere) with radius r and mass m with a cue on a pool table. The collision is quick and you are hitting the ball at height h, and the middle, i.e. not left or right with respect to the center (the problem is two dimensional with fixed axis of rotation). Your cue has a force sensor which integrates the force in the collision time and give you the result, the impulse J. This impulse is horizontal.

a) Find the initial velocity and angular velocity of the ball, right after the collision.
b) Call the initial velocity and angular velocities v_0 and ω_0, as shown in fig. 3. Explain the motion of the ball. The coefficient of the kinetic friction is μ_k. How long it takes for the ball to start rolling without any sliding? What is the final velocity?
c) [fun] If you hit the ball in way that the initial velocity was $\mathbf{v}_0$ and initial angular velocity was $\boldsymbol{\omega}_0$ in arbitrary directions, find the path of the ball on the table (three dimensional problem).

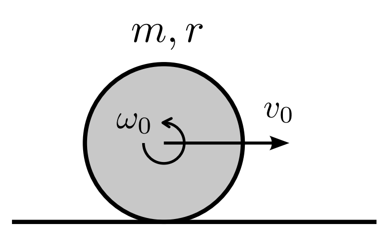

Figure 3: Initial velocity and angular velocity of a ball.

4) A uniform wooden log with mass M and length L is sitting on a frictionless ice. We shoot a bullet with mass m and velocity v perpendicular to the log, to a distance b from the center of the log. See fig. 4. This bullet sticks to the log.
a) Which observables, for the system of the bullet and the log, are conserved in this collision? Write down the equations.
b) Solve the equations and explain the motion of the system after the collision.
c) [fun] Explain the motion of the system after the collision, if there is friction with coefficient of the kinetic friction being μ_K. The collision was instantaneous.

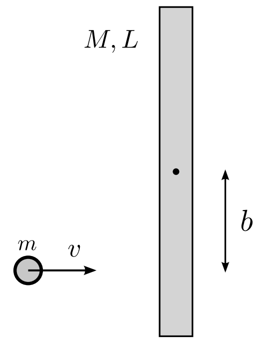

Figure 4: A bullet hitting a log.

5) You are playing baseball with a uniform wooden rod instead of a bat. The length of the rod is L and you hold the rod from one end. The ball hits the rod at the length b from the end that you are holding it. Assume that the ball give the rod an impulse J, perpendicular to the rod. During the collision, your hand gives an impulse to the rod which we call J_h perpendicular to the rod. See fig. 5.
a) Find J_h in terms of J.
b) Set $J_h = 0$ and find b for the "center of percussion". The pivot becomes the instantaneous center of rotation. If you hit the ball correctly your hand does not feel any impulse.
c) [fun] Find the specifications of a bat and find the center of percussion for a real bat.

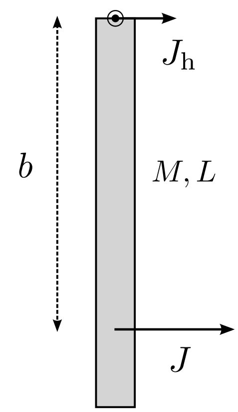

Figure 5: A rod with the pivot (hand) on top.

6) Consider a spool with inner radius r, outer radius R, mass M, and moment of inertia I. This spool rolls on the ground without slipping. We connect this spool to a hanging mass m over a massless frictionless pulley. See fig. 6.
a) Draw the forces and discuss about the direction of the static friction force.
b) Find the tension force, the acceleration of the spool, and the acceleration of the hanging mass.

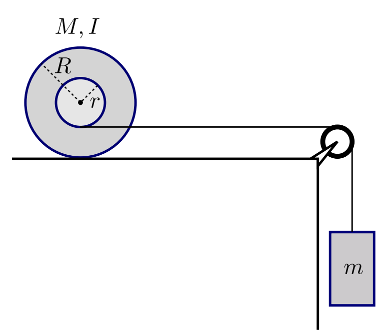

Figure 6: A spool connected to a hanging mass.

7) Consider a yo-yo sitting on the ground, with inner radius r, outer radius R, mass M, and moment of inertia I. You are pulling the rope with an angle θ with a force F. Discuss the motion of the yo-yo in all possible scenarios. For any value of F, any $-\pi < \theta \leq \pi$, and when μ_S is infinite or μ_S is finite. You are pulling the rope in a manner that θ stays constant. See fig. 7, where six examples of θ are shown. Why are the angles $\pm\cos^{-1}(r/R)$ interesting?

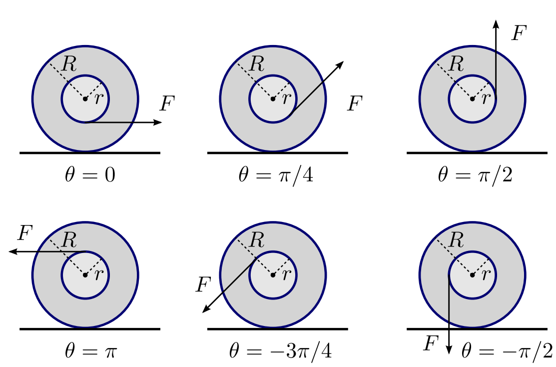

Figure 7: A yo-yo.