Physics 271, Section . . . Thursday, Nov 20, 2014
Quiz #11 Oscillations

Name: [5 pts]

1) A uniform rad with mass m and length $r_1 + r_2$ is pivoted at a point. Ignore gravity or assume all the motions are happening on a horizontal plane, i.e. looking from above systems looks like fig. 1. At a distance r_1 from this rod, there is a spring connected to rod with constant k_1, and at a distance r_2 there is another spring connected to the rod with constant k_2. See fig. 1. All the motion can be explained by one degree of freedom θ. Also assume $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.

a) In terms of θ, find the spring forces. **[2 pts]**
b) Find the torque, about the pivot point, in terms of θ. **[3 pts]**
c) Prove that $I_p = m(r_1^2 + r_2^2 - r_1 r_2)/3$ where p is the pivot point. **[2 pts]**
d) Find the frequency of the small oscillations. **[3 pts]**
e) Write down a formula for the mechanical energy, in terms of θ and $\dot{\theta}$. What is the effective "mass" and effective "spring constant" for the parameter θ? **[2 pts]**

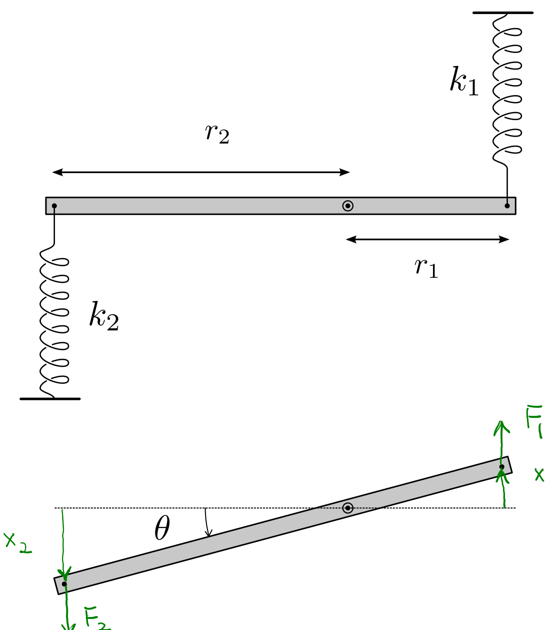

Figure 1: A rod pivoted to a point and connected to springs. I drew the rod again at an angle θ.

a) $x_1 = r_1\theta$ $\qquad F_1 = -k_1 x_1 = -k_1 r_1 \theta$

$x_2 = r_2\theta$ $\qquad F_2 = -k_2 x_2 = -k_2 r_2 \theta$

b) $$\tau = F_1 r_1 + F_2 r_2 = -(k_1 r_1^2 + k_2 r_2^2)\,\theta$$

c) Parallel axis th'm:

$$I_{cm} = \frac{1}{12} m (r_1 + r_2)^2$$

$$I_p = I_{cm} + m\left(\frac{r_1 - r_2}{2}\right)^2 = m\left[\frac{r_1^2 + r_2^2 + 2r_1r_2}{12} + \frac{r_1^2 + r_2^2 - 2r_1r_2}{4}\right]$$

$$= m\,\frac{4r_1^2 + 4r_2^2 - 4r_1r_2}{12} = \frac{1}{3} m (r_1^2 + r_2^2 - 2r_1r_2).$$

d) $$\tau = I_p\ddot{\theta} \rightarrow -(k_1r_1^2 + k_2r_2^2)\theta = I_p\ddot{\theta} \rightarrow \ddot{\theta} + \frac{k_1r_1^2 + k_2r_2^2}{I_p}\theta = 0$$

so $$\omega = \left[\frac{k_1r_1^2 + k_2r_2^2}{\frac{1}{3}m(r_1^2 + r_2^2 - r_1r_2)}\right]^{1/2}.$$

e) $$E = U + K = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}I_p\dot{\theta}^2 = \frac{1}{2}(k_1r_1^2 + k_2r_2^2)\theta^2 + \frac{1}{2}I_p\dot{\theta}^2$$

$$K_{eff} = k_1r_1^2 + k_2r_2^2, \qquad M_{eff} = I_p.$$