Physics 271, Section ... Thursday, Sep 18, 2014
Quiz #2 Vectors, 2D & 3D Kinematics

Name:

1) You are playing soccer. You are the goalkeeper and you are at the origin, and the field is the xy plane, as shown in the fig. 1. You want to shoot the ball with a velocity vector $\mathbf{v}_0 = (v_{0x}, v_{0y}, v_{0z})$ so that it reaches the midfield player at $\mathbf{r} = (96 \text{ m}, 40 \text{ m}, 0)$ the exact time the player gets there. The ball travels through the air in one projectile motion. The player is running with velocity $v = 4$ m/s, toward $\mathbf{r}$ from $\mathbf{r}_0 = (66 \text{ m}, 0, 0)$.

a) Find the distance $|\mathbf{r} - \mathbf{r}_0|$. **[1 pt]**
b) How long it takes for the player to get from $\mathbf{r}_0$ to $\mathbf{r}$? **[1 pt]**
c) One the figure, draw the velocity vector $(v_{0x}, v_{0y}, 0)$. What is v_{0y}/v_{0x}? Describe the ball's motion on xy plane, ignoring the z direction. **[2 pts]**
d) What is the distance $|\mathbf{r}|$? **[2 pts]**
e) Find v_{0x} and v_{0y}. **[2 pts]**
f) Explain the ball's motion in the z direction. Find v_{0z}. **[2 pts]**
g) What is the initial angle, θ_0, of this projectile motion? **[2 pts]**

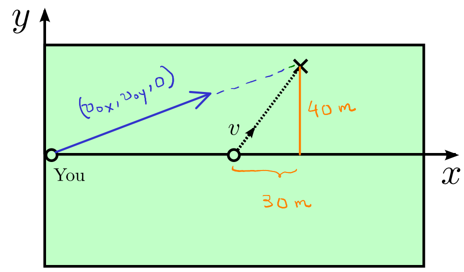


Figure 1: Long distance pass.

a) $\left.\begin{array}{l}\vec{r} = 96\,\hat{x} + 40\,\hat{y} + 0\,\hat{z} \\ \vec{r}_o = 66\,\hat{x} + 0\,\hat{y} + 0\,\hat{z}\end{array}\right\} \quad \vec{r} - \vec{r}_o = 30\hat{x} + 40\hat{y} + 0\,\hat{z} \rightarrow |\vec{r} - \vec{r}_o| = \sqrt{30^2 + 40^2} = 50 \text{ m}.$

b) The player is running $|\vec{r} - \vec{r}_o| = 50$ m with constant velocity $v = 4$ m/s so $t_f = \dfrac{50^m}{4\frac{m}{s}} = \dfrac{25}{2}$ sec.

$$\begin{cases} x_p(t) = 66 + \frac{12}{5} t \\ y_p(t) = \frac{16}{5} t \end{cases}$$

c) In xy plane, the acceleration is zero, so the ball goes straight line, parallel to $\vec{r}$.

So $\dfrac{v_{oy}}{v_{ox}} = \dfrac{r_y}{r_x} = \dfrac{40 \text{ m}}{96 \text{ m}} = \dfrac{5}{12}$. $\quad \begin{cases} x_b(t) = v_{ox} t \\ y_b(t) = v_{oy} t = \frac{5}{12} v_{ox} t \end{cases}$

d) $|\vec{r}| = \sqrt{96^2 + 40^2} = 8 \cdot 13 = 104$ m.

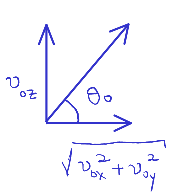
e) At exact time $t_f = \frac{25}{2}$ sec the ball must reach $\vec{r}$. So $x_b(t_f) = v_{ox} t_f = 96$ m,

or $v_{ox} = \dfrac{192}{25} \dfrac{m}{s} \rightarrow v_{oy} = \dfrac{5}{12} \cdot \dfrac{192}{25} = \dfrac{16}{5} \dfrac{m}{s}$.

f) Constant acceleration $a_z = -g$. $\quad z(t) = 0 + v_{oz} t - \frac{1}{2} g t^2$.

$z(t_f) = 0 \rightarrow v_{oz} = \frac{1}{2} g \left(\frac{25}{2}\right) = \frac{125}{2}$ m/s.

g)



$$\tan \theta_o = \frac{v_{oz}}{\sqrt{v_{ox}^2 + v_{oy}^2}} = \frac{125/2}{208/25} = \frac{5^5}{416} \approx 7.5 \rightarrow \theta_o \approx 80^\circ.$$