

Name: [5 pts]

1) Consider three masses as shown in the fig. 1, the masses m_1 and m_2 at a horizontal plane, connected over a pulley to a mass M on an inclined plane with an angle θ . Use numerical values $m_1 = 2$ kg, $m_2 = 3$ kg, $M = 5$ kg, and $\theta = \sin^{-1}(12/13)$, if you want.

a) Say you hold the mass m_2 with a force F so that the system is at equilibrium. Find F and the tension forces of both strings. [3 pts]

b) You let the system go. How are the accelerations of these masses related to each other? Show the acceleration vectors on the figure. [2 pts]

c) Write down the equations of motion for each mass. [4 pts]

d) Find the accelerations and tension forces. [3 pts]

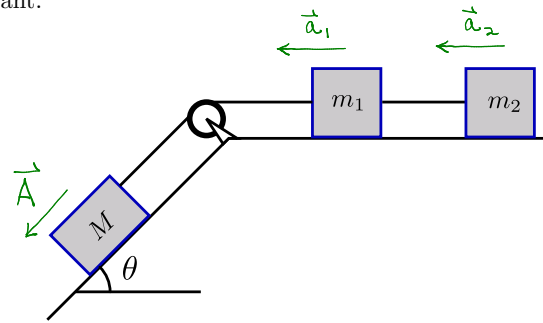
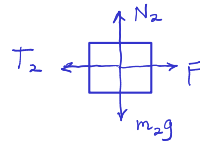


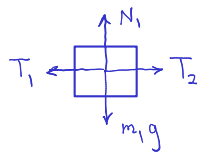
Figure 1: Three masses.

a) equilibrium or " $\sum F = 0$ "

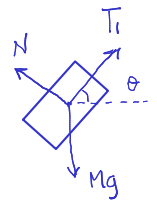
$$m_2: \begin{cases} N_2 - m_2 g = 0 \\ T_2 - F = 0 \text{ (i)} \end{cases}$$



$$m_1: \begin{cases} N_1 - m_1 g = 0 \\ T_1 - T_2 = 0 \text{ (ii)} \end{cases}$$



$$M: \begin{cases} N - Mg \cos \theta = 0 \\ Mg \sin \theta - T_1 = 0 \text{ (iii)} \end{cases}$$

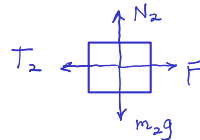


$$(i), (ii), \text{ \& } (iii) \Rightarrow \boxed{F = T_1 = T_2 = Mg \sin \theta}$$

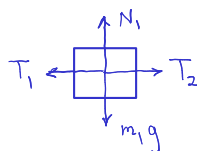
b) $\vec{a}_1 = \vec{a}_2$, $|\vec{A}| = |\vec{a}_1| = |\vec{a}_2| =: a$ The length of the strings are constant.

c) " $\sum F = ma$ "

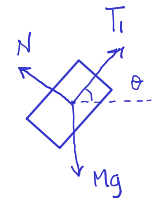
$$m_2: \begin{cases} N_2 - m_2 g = 0 \\ T_2 = m_2 a \text{ (i)} \end{cases}$$



$$m_1: \begin{cases} N_1 - m_1 g = 0 \\ T_1 - T_2 = m_1 a \text{ (ii)} \end{cases}$$



$$M: \begin{cases} N - Mg \cos \theta = 0 \\ Mg \sin \theta - T_1 = Ma \text{ (iii)} \end{cases}$$



$$d) (i), (ii), \text{ \& } (iii) \Rightarrow \boxed{a = \frac{Mg \sin \theta}{m_1 + m_2 + M}} \rightarrow \boxed{T_2 = \frac{m_2 Mg \sin \theta}{m_1 + m_2 + M}}, \boxed{T_1 = \frac{(m_1 + m_2) Mg \sin \theta}{m_1 + m_2 + M}}$$