Name: [5 pts]

1) A mass m is connected to a spring with spring constant k in a horizontal plane. On the opposite side this mass is connected to a hanging mass M using a pulley. See fig. 1. We are holding the mass m where the spring is relaxed, $x = 0$. The whole system can be explained by the position of the mass m, which we will call x. If m moves in positive direction, spring stretches and M goes down.

a) What is the potential energy stored in the spring in terms of x? **[2 pts]**
b) What is the gravitational potential energy at x? **[2 pts]**
c) Write down the total potential energy, $U(x)$, in terms of x. Draw $U(x)$ vs x in a graph. **[2 pts]**
d) Say we release m from $x = 0$. Show the total energy level on the above graph. **[1 pt]**
e) Find maximum velocity that the masses reach. **[2 pts]**
f) Find how much the mass M goes down till it stops and starts coming back. **[2 pts]**
g) Do parts e and f again, assuming the coefficient of kinetic friction between m and the surface is μ_k. Explain the motion on the energy graph. **[4 pts]**

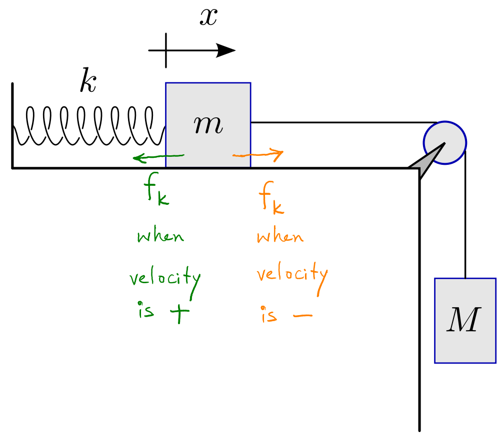


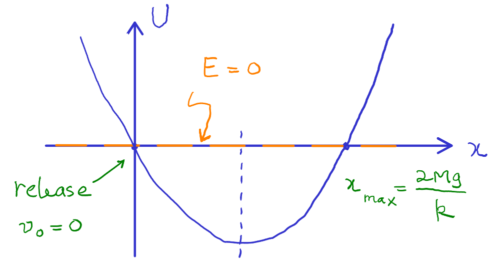
Figure 1: The spring and gravity.

a) $U_{spring} = \frac{1}{2}kx^2$

b) $U_g = -Mgx + \cancel{U_0}$ I set it to zero

c) $U = U(x) = \frac{1}{2}kx^2 - Mgx$

d) $E = U(0) + K_i = 0$.



$x^* = \frac{Mg}{k}$, $U(x^*) = \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 - Mg\left(\frac{Mg}{k}\right) = -\frac{(Mg)^2}{2k}$

e) The masses reach max velocity when

$$\left.\frac{dv}{dt}\right|_{v=v_{max}} = 0 \quad \text{or} \quad \left.F\right|_{v=v_{max}} = 0,$$

which is the bottom point in the graph.

$$F(x) = -\frac{dU}{dx} = -kx + Mg, \quad F(x^*) = 0 \rightarrow x^* = \frac{Mg}{k}.$$

So let's write energy conservation law for the system,

$$\begin{cases} i: & x = 0 \\ f: & x = Mg/k \end{cases} \quad E_i = E_f \Rightarrow 0 = \frac{1}{2}(m+M)v_{max}^2 - \frac{(Mg)^2}{2k} \rightarrow v_{max} = Mg\sqrt{\frac{1}{k(m+M)}}.$$

f) At x_{max}, KE $= 0$, so $U(x_{max}) = E = 0 \rightarrow \frac{1}{2}kx^2 - Mgx = 0 < \begin{matrix} x = 0. \\ x_{max} = \frac{2Mg}{k}. \end{matrix}$

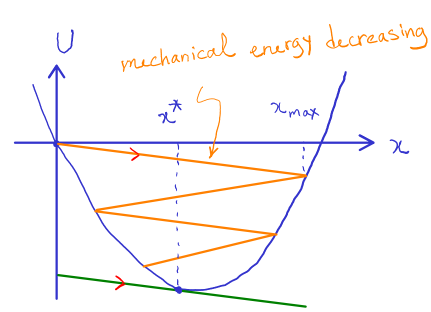
$f_k = \mu_k mg$

g) 'e'. $F(x) = -f_k - kx + Mg$, $F(x^*) = 0 \rightarrow x^* = \frac{Mg - \mu_k mg}{k}$.

$$\begin{cases} i: & x = 0 \\ f: & x = x^* \end{cases} \quad W_{friction} = -f_k(x^* - 0) = -\mu_k mg\, x^* = E_f - \cancel{E_i}^{\,0} = \frac{1}{2}(m+M)v_{max}^2 + U(x^*)$$

$$\rightarrow v_{max} = (Mg - \mu_k mg)\sqrt{\frac{1}{k(m+M)}}.$$

if $v_{max} \le 0 \rightarrow$ no movement (static)



'f'. At x_{max}, KE $= 0$, so $W_{friction} = U(x_{max})$

$$\rightarrow -f_k x_{max} = \frac{1}{2}kx_{max}^2 - Mgx_{max} \rightarrow x_{max} = \frac{2(Mg - \mu_k mg)}{k}.$$