Physics 271, Section ... Quiz #7 | Thursday, Oct 23, 2014 Linear Momentum

Name: [5 pts]

1) We cut out a right-angled triangle from a square with side length 1. The vertices of this cut triangle is at $\{(1/2, 1/2), (1, 0), (1, 1)\}$. See fig. 1. We want to find the center of mass, $(x_{\text{CM}}, y_{\text{CM}})$, of this uniform surface, i.e. the mass per unit area is constant.

a) Find y_{CM}. **[2 pts]**

b) Find x_{CM}. **[4 pts]**

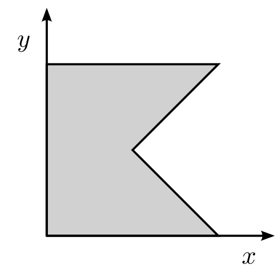


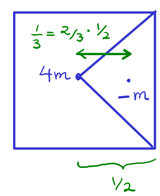
Figure 1: Where is the center of mass?

a) $y_{CM} = 1/2$. There is a reflection symmetry here, the line is $y = 1/2$.

b) method i.

Square w/ mass $4m$

Cut triangle w/ mass $-m$



$$x_{CM} = \frac{1}{M} \sum m_i x_i$$

$$= \frac{1}{4m - m} \left[4m \cdot \frac{1}{2} - m \cdot \left(\frac{1}{2} + \frac{1}{3} \right) \right] = \frac{7}{18}.$$

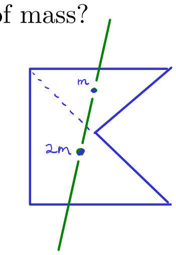
method ii.

Big triangle at $\left(\frac{1}{3}, \frac{1}{3}\right)$

Small triangle at $\left(\frac{1}{2}, \frac{5}{6}\right)$

Line passing through them is $y = 3x - 2/3$

this intersects w/ the line $y = \frac{1}{2}$ at $3x_{CM} - 2/3 = 1/2 \Rightarrow x_{CM} = 7/18$.



Of course there are other ways to calculate x_{CM}.

2) A mass m is moving with velocity v_0. In front of this mass there are two similar masses m connected together by a spring, initially at rest. See fig. 2. The mass on the left collides with the mass in the middle. The collision is elastic and hard (instantaneous).

→ mass 2 position does not change right after the collision

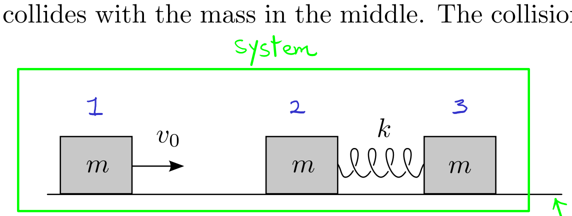


Figure 2: A moving mass spring system.

ground is not a part of system

a) What are the velocities for each one of the masses, right after the collision? **[3 pts]**

b) What is the velocity of the center of mass of this system? Does it change? **[2 pts]**

c) What is the maximum amount that the spring compresses? What is the maximum amount that it stretches? **[3 pts]**

a) 1 stops, $v_1 = 0$; $v_2 = v_0$; $v_3 = 0$.

b) $x_{CM} = \frac{1}{3}(x_1 + x_2 + x_3) \rightarrow v_{CM} = \frac{1}{3}(v_1 + v_2 + v_3) = \frac{v_0}{3}$.

No. The only external forces are normal forces and gravitational forces, which cancel out. So $\vec{F}_{ext} = M \frac{d\vec{v}_{CM}}{dt} = 0 \rightarrow \vec{v}_{CM} = \frac{v_0}{3} \hat{x}$.

$$\begin{cases} \vec{F}_{ext} \cdot \hat{y} = 3N - 3mg = 0, \\ \vec{F}_{ext} \cdot \hat{x} = 0. \end{cases}$$

c) $i: v_2 = v_0,\ v_3 = 0$

$f: v_2 = v_3 = v$

when it's maximally compressed or stretched

$P_i = P_f \rightarrow m v_0 = m v + m v \rightarrow v = \frac{v_0}{2}$

$E_i = E_f \rightarrow U_i + K_i = U_f + K_f \rightarrow 0 + \frac{1}{2} m v_0^2 = \frac{1}{2} k x^2 + \frac{1}{2} m \left(\frac{v_0}{2}\right)^2 + \frac{1}{2} m \left(\frac{v_0}{2}\right)^2$

$\rightarrow x = v_0 \sqrt{\frac{m}{2k}}$.