Physics 271, Section ...
Quiz #8

Thursday, Oct 30, 2014
Rotation

Name: [5 pts]

1) moment of inertia
Consider a shape of the letter H, as shown in the fig. 1, consists of three uniform rods, with length l and mass m, welded together.
a) Find I_x. **[2 pts]**
b) Find I_y. **[2 pts]**
c) Find I_z. **[2 pts]**

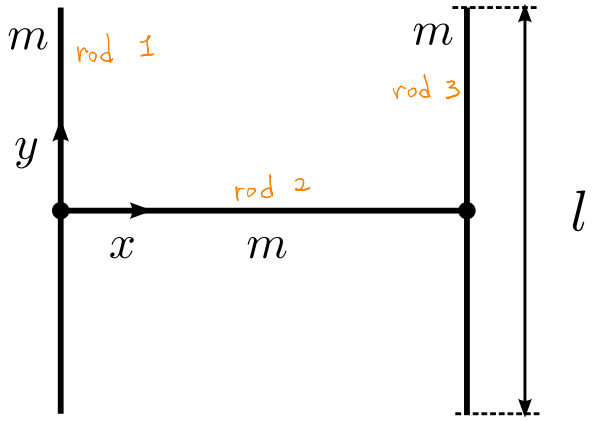

Figure 1: What is the moment of inertia?

a) $I_x = I_{x_1} + I_{x_2} + I_{x_3}$
$= \frac{1}{12}ml^2 + 0 + \frac{1}{12}ml^2 = \frac{1}{6}ml^2$

b) $I_y = I_{y_1} + I_{y_2} + I_{y_3}$
$= 0 + \frac{1}{3}ml^2 + ml^2 = \frac{4}{3}ml^2$

c) $I_z = I_{z_1} + I_{z_2} + I_{z_3}$
$= \frac{1}{12}ml^2 + \frac{1}{3}ml^2 + \left(\frac{1}{12}ml^2 + ml^2\right) = \frac{3}{2}ml^2 = I_x + I_y$.

2) A yo-yo is falling down as shown in the fig. 2. This yo-yo has a radius r, mass M, and the moment of inertia about the axis perpendicular to circles, passing through the center of it, is I.

a) Find a relation between the velocity of the center of the yo-yo and the angular velocity of the yo-yo. **[1 pt]**
b) Write down the kinetic energy in terms of r, M, I, and v, the velocity of the center. **[2 pts]**
c) Using energy conservation, find the velocity of the yo-yo after it release it from rest and it comes down a height h. **[2 pts]**

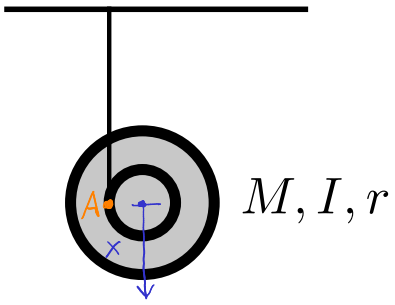

Figure 2: A yo-yo falling down.

a) After coming down x, yo-yo has to rotate $\theta = \frac{x}{r}$ to open length x of the rope.
So $x_{CM} = r\theta \rightarrow v_{CM} = r\omega$.

b) $K = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(Mr^2 + I)\omega^2$.
or $K = \frac{1}{2}M\cancel{v_A^2}^{0} + \frac{1}{2}I_A\omega^2 = \frac{1}{2}(I + Mr^2)\omega^2$.

c) $E_i = Mgh + 0$
$E_f = 0 + K$
$\rightarrow \frac{1}{2}(I + Mr^2)\omega^2 = Mgh \rightarrow \omega = \sqrt{\frac{2gh}{\left(r^2 + \frac{I}{M}\right)}}$.