Physics 271, Section ... Thursday, Nov 6, 2014
Quiz #9 Rolling, Torque, & Angular Momentum

Name: [5 pts]

1) A ball with mass m and initial velocity v_0, hits a uniform rod with mass M and length L with a pivot at its center. The collision is elastic and hard (instantaneous) and happens at a distance b from the pivot. And the ball initially was moving perpendicular to the rod. See fig. 1. Take the rod and the ball to be the system. Remember that there is an external force acting on this system, namely the pivot's force on the rod.

a) From the values, total kinetic energy, total momentum, and total angular momentum around the pivot, which ones are conserved, before and after the collision? Explain in one sentence why. By total I mean the values for the system. **[6 pts]**
b) Write down the equations for the conserved quantities before and after the collision. Please use the subscript i and f for before and right after the collision. **[4 pts]**
c) Find the final velocity of the ball and angular velocity of the rod. **[2 pts]**
d) Find the impulse that pivot experience in this collision. **[2 pts]**
e) [fun] Find how many times the ball will collide to the rod.

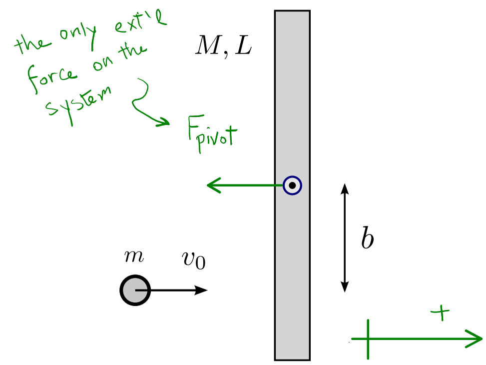

Figure 1: A rod fixed with a pivot, and the ball before hitting the rod.

a) The collision is elastic & the work done by F_{pivot} is zero. so total kinetic energy is conserved.
The external force F_{pivot} can have impulse : $J_p = \int F_{pivot}\, dt$ So the total momentum is not conserved.
The external Torque is zero as the Lever arm for F_{pivot} around pivot is zero So the total angular momentum is conserved.

b) $K_i = \frac{1}{2} m v_0^2 = K_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2$ (i) $\quad I = \frac{1}{12} M L^2$

$L_i = b m v_0 = L_f = b m v_f + I \omega_f$ (ii)

c) $v_0 - v_f = \frac{I}{bm} \omega_f$ (ii')

$\frac{1}{2} m (v_0^2 - v_f^2) = \frac{1}{2} I \omega_f^2 \rightarrow \frac{1}{2} m (v_0 - v_f)(v_0 + v_f) = \frac{1}{2} I \omega_f^2 \xrightarrow{(ii')} v_0 + v_f = b\omega_f$ (i')

$$\rightarrow \begin{cases} \omega_f = \dfrac{v_0}{b} \dfrac{2mb^2}{I + mb^2} \\ v_f = v_0 \dfrac{mb^2 - I}{I + mb^2} \end{cases}$$

Check the answer for $b=0$.
Also the case when $I = mb^2$.

d) $P_i = m v_0$, $P_f = m v_f$ (The rod's center of mass is not moving)

$J_p = P_f - P_i = m(v_f - v_0) = m v_0 \left(\dfrac{-2I}{I + mb^2} \right)$.