Phys 124 @ Rutgers

tutorial #2

equilibrium & elasticity

Problem set #1

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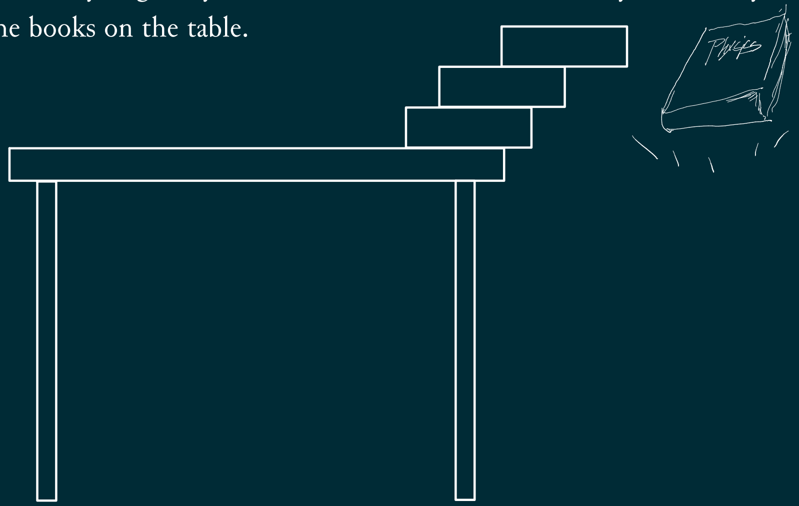
Problem #1

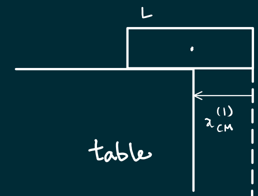
There are bunch of physics books sitting on the table. Can you make a book balanced in a way that no part of the book is above the table? How far can you get if you have N books? Remember that you can only use the books on the table.



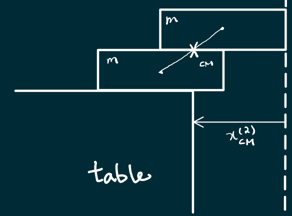
$$\vec{R}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

$$M = \sum_{i=1}^{N} m_i$$

1) $x_{CM}^{(1)} = \frac{L}{2}$

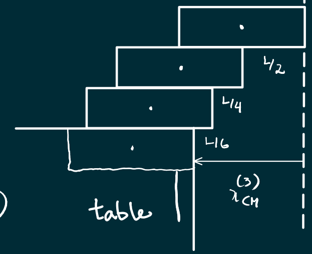


2) $x_{CM}^{(2)} = \frac{1}{2m}\left(m\frac{L}{2} + mL\right) = \frac{3}{4}L$



3) $x_{CM}^{(3)} = \frac{1}{3m}\left(m\frac{L}{2} + mL + m\frac{5}{4}L\right)$

$= \frac{11}{12}L$



N) $x_{CM}^{(N)} = \frac{L}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N}\right)$

$= \frac{L}{2} H_N .$

this is true for N, show that it's true for N+1.

$$x_{CM}^{(N+1)} = \frac{1}{(N+1)m}\left[Nm\frac{L}{2}H_N + m\left(\frac{L}{2}H_N + \frac{L}{2}\right)\right]$$

$= \frac{L}{2}H_{N+1}$. $\Rightarrow$ We proved that $x_{CM}^{(N)} = \frac{L}{2}H_N$.

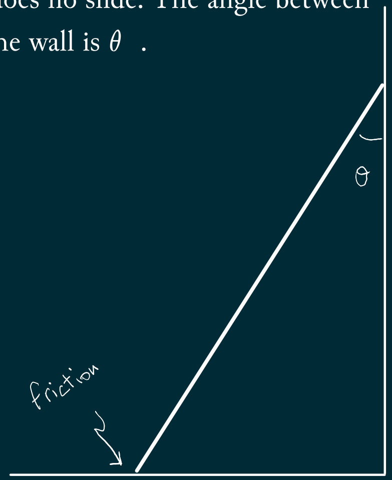
exercise 2.a find $x_{CM}^{(4)}$ and show that the last book is after the edge of the table.

Adding to the video:
As you know harmonic series is divergent so you can get infinitely far from the edge of the table as long as you have enough number of books. But it grows really slowly like natural log function.

Correction to the video:
During explaining exercise at some moment I say 24/25, but 25/24 = 1/2+1/4+1/6+1/8 > 1 is correct.

Problem #2

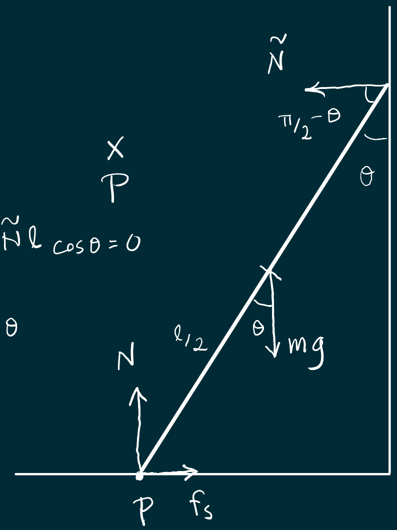
A uniform ladder with mass m and length l is leaning against a vertical wall. There is no friction between the ladder and this wall but there is friction between the ladder and ground. Find the minimum of static friction coeff. so that the ladder does no slide. The angle between the ladder and the wall is θ .



- forces
- equilibrium conditions
- $f_s \le \mu_s N$

$\sum \vec{F} = 0$: $\hat{x}$: $f_s = \tilde{N}$; $\hat{y}$: $N = mg$

$\sum \tau_P = 0 \rightarrow -mg\frac{l}{2}\sin\theta + \tilde{N} l \cos\theta = 0 \rightarrow \tilde{N} = \frac{mg}{2}\tan\theta$



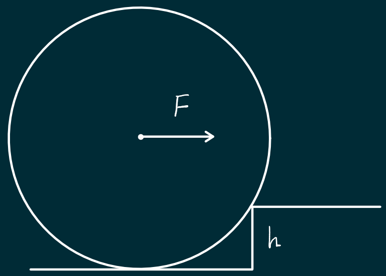
$$f_s = \frac{mg}{2}\tan\theta \le \mu_s N$$

$$\mu_s \ge \frac{f_s}{N} = \frac{1}{2}\tan\theta$$

$$\min(\mu_s) = \frac{1}{2}\tan\theta .$$

Problem #3

Find the minimum horizontal force F to take up a wheel with mass m and radius R from a bump with height h. The force is acting on the center of the wheel.



- forces

- conditions (equilibrium ?)

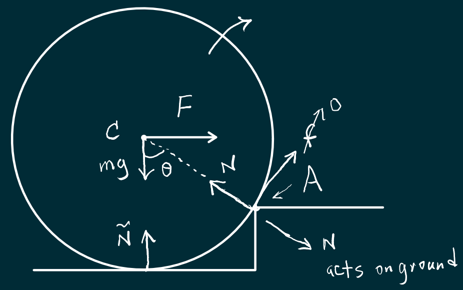
$$\sum \vec{F} = 0 \quad \begin{cases} F - N\sin\theta + \underbrace{\cancel{f}}_{0}\cos\theta = 0 & (1) \\ -mg + \tilde{N} + N\cos\theta + \underbrace{\cancel{f}}_{0}\sin\theta = 0 & (2) \end{cases}$$

$$\sum \tau_p = 0 \; - \; \tau_c = 0 \to fR = 0 \to f = 0$$

Condition : $\tilde{N} = 0$

$$\overset{(1),(2)}{\hookrightarrow} \; F = mg\tan\theta$$

$$= mg\,\frac{\sqrt{R^2 - (R-h)^2}}{R-h} = mg\left[\left(\frac{R}{R-h}\right)^2 - 1\right]^{1/2}.$$



Adding to the video:
Have fun with the result and try to understand limiting points. What happens when h=0? How about R=h. This is a good practice for testing your answer, i.e. considering the limit points.

Correction to the video:
At some point calculating the last line, tanθ, I say circle instead of triangle.