the 1st law of thermodynamics, problem set #2

1) The compression ratio of a diesel engine is 15 to 1; that is, air in a cylinder is compressed to 1/15 of its initial volume. Use the values $C_V = 20.8$ J/mol · K and $\gamma = 1.40$ for air.
a) If the initial pressure is 1.0×10^5 Pa and the initial temperature is 27°C, find the final pressure and the temperature after adiabatic compression.
b) How much work does the gas do during the compression if the initial volume of the cylinder is 1.0×10^{-3} m^3?

a) in an adiabatic process: $P_i V_i^{\gamma} = P_f V_f^{\gamma}$ and $\frac{V_f}{V_i} = \frac{1}{15} \rightarrow P_f = P_i \left(\frac{V_i}{V_f}\right)^{\gamma} = 45 \times 10^5$ Pa.

state eqn $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$ " " $\rightarrow T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = 890$ K.

b) for an adiabatic process $W = -\Delta U = n C_V (T_i - T_f) = \left(\frac{P_i V_i}{R T_i}\right) C_V (T_i - T_f)$

$= -.49$ kJ.

2) The graph in fig. 1 shows a pV-diagram of the air in a human lung when a person is inhaling and then exhaling a deep breath. Such graphs, obtained in clinical practice, are normally somewhat curved, but we have modeled one as a set of straight lines of the same general shape. (The pressure shown is the gauge pressure, not the absolute pressure.)

a) How many joules of net work does this person's lung do during one complete breath?
The pressure change is due to changes in the amount of gas in the lung, not to temperature changes. Think of your own breathing. Your lungs do not expand because they've gotten hot.
b) If the temperature of the air in the lung remains a reasonable 20°C, what is the maximum number of moles in this person's lung, n_{max}, during a breath? Which point in the graph corresponds to this n_{max}?
c) If the temperature only changes from 20°C to 37°C estimate the change in n_{max} in part 'b'.

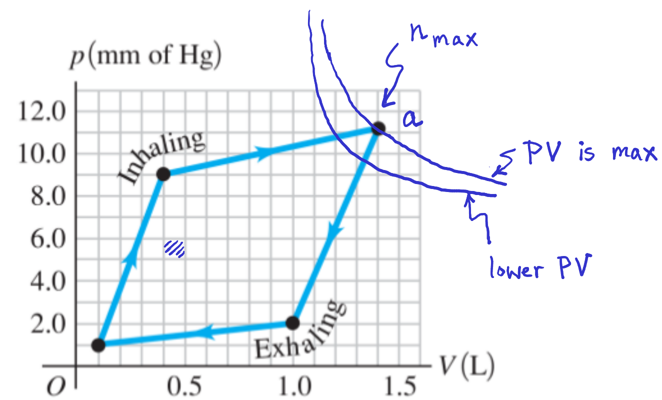

Figure 1: pV-diagram.

a) W done in a cycle is equal to the area captured in pV-diagram,
each square is 1 mmHg · 0.1 L $= \frac{1.0 \times 10^5}{760}$ Pa · 0.1×10^{-3} m^3 $= 13 \times 10^{-3}$ J
$\rightarrow$ W = 1.0 J > 0 as the cycle goes clockwise.

b) When inhaling finished, at point a in the diagram, $n_a = n_{max} = \frac{P_a V_a}{RT} = .06$ mol.

c) using T = 310 K instead of T = 293 would not change the $n_{max} = \left(\frac{PV}{RT}\right)_{max}$ point, and one can still estimate using point a.

if n is changing, then we need to know the processes in one other diagram, either one of VT, pT, nT, np, nV would work. in part b we easily said T = const.

3) The graph in fig. 2 shows a pV-diagram for $n = 3.25$ mol of ideal helium, He, gas. Part ca of this process is isothermal.

monatomic ideal gas

$C_p = 5/2\, R\,,\ C_V = \frac{3}{2} R$

a) Find the pressure of the He at point a.
b) Find the temperature of the He at points a, b, and c.
c) How much heat entered or left the He during segments ab, bc, and ca? In each segment, did the heat enter or leave?
d) By how much did the internal energy of the He change from a to b, from b to c, and from c to a? Indicate whether this energy increased or decreased.

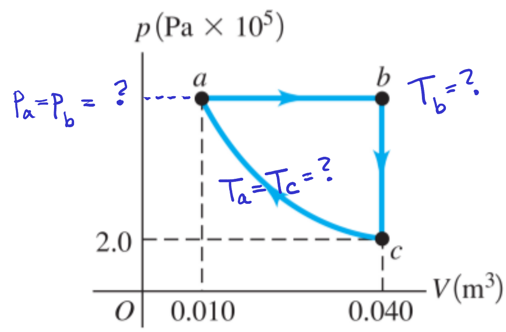

Figure 2: pV-diagram for He gas.

$n = 3.25$ and const.

before doing these kind of problems, it is a good practice to write down quick info about pVT for each point from the diagram, specilly when the processes are iso-something.

all info in the diagram ; it is a cycle abc &

$P_a = P_b$ (i) | $P_c = 2.0 \times 10^5$ Pa

$V_b = V_c$ (ii) | $V_c = 0.040\ m^3$

$T_a = T_c$ (iii) | $V_a = .010\ m^3$

a, b) now look to see if in any point we are missing one variable in $PV = nRT$

$$P_c V_c = nRT_c \rightarrow T_c = 296\ K = T_a$$

$$P_a = \frac{nRT_a}{V_a} = 8.0 \times 10^5\ Pa = P_b$$

$$T_b = \frac{P_b V_b}{nR} = 1184\ K.$$

c) ab is isobaric: $Q_{ab} = n\,C_p\,(T_b - T_a) = 60\ kJ > 0$, given to He gas or enters

bc is isochoric: $Q_{bc} = \Delta U_{bc} = n C_V (T_c - T_b) = -36\ kJ < 0$, leaves the gas

ca is isothermal: $Q_{ca} = W_{ca} = nRT_a \ln\left(\frac{V_a}{V_c}\right) = -11\ kJ < 0$, " " "

d) ab is isobaric: $\Delta U_{ab} = n C_V (T_b - T_a) = 36\ kJ > 0$, increased

bc is isochoric: $\Delta U_{bc} = Q_{bc} = -36\ kJ < 0$, decreased

ca is isothermal: $\Delta U_{ca} = 0$.