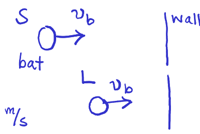
beats and interference, problem set #2

1) A bat flies toward a wall, emitting a steady sound of frequency $f_0 = 1.70$ kHz. This bat hears its own sound plus the sound reflected by the wall. How fast should the bat fly in order to hear a beat frequency of $f_{\text{beat}} = 20.0$ Hz?

bat as Source : $f_1 = \frac{v}{v - v_b} f_0$

bat as Listener: $f_2 = \frac{v + v_b}{v} f_1 = \frac{v + v_b}{v - v_b} f_0$

$\rightarrow f_{\text{beat}} = |f_2 - f_0| = \frac{2 v_b}{v - v_b} f_0 \rightarrow v_b = v \frac{f_{\text{beat}}}{2 f_0 + f_{\text{beat}}} = 340 \frac{m}{s} \frac{20.0 \text{ Hz}}{3.40 \text{ kHz}} = 2.0 \text{ m/s}$



2) A woman stands at rest in front of a large, smooth wall. She holds a vibrating tuning fork of frequency f_0 directly in front of her (between her and the wall).
a) The woman now runs toward the wall with speed v_{W}. She detects beats due to the interference between the sound waves reaching her directly from the fork and those reaching her after being reflected from the wall. How many beats per second will she detect? (Note: If the beat frequency is too large, the woman may have to use some instrumentation other than her ears to detect and count the beats.)
b) If the woman instead runs away from the wall, holding the tuning fork at her back so it is between her and the wall, how many beats per second will she detect?

Similar to problem 1. In part b, $f_2 = \frac{v - v_W}{v + v_W} f_0$ so $f_{\text{beat}} = \frac{2 v_W}{v + v_W} f_0$. usually $v_W \ll v$ so $f_{\text{beat}} \approx \frac{2 v_W}{v} f_0$.

3) Two identical loudspeakers are located at points A and B, $d = 2.00$ m apart as shown in fig. 1. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of $f = 880$ Hz. Take the speed of sound in air to be $v_s = 340$ m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B.

a) At what distances from B will there be destructive interference?
b) At what distances from B will there be constructive interference?
c) If the frequency is made low enough, there will be no x at which destructive interference occurs. How low must the frequency be for this to be the case?

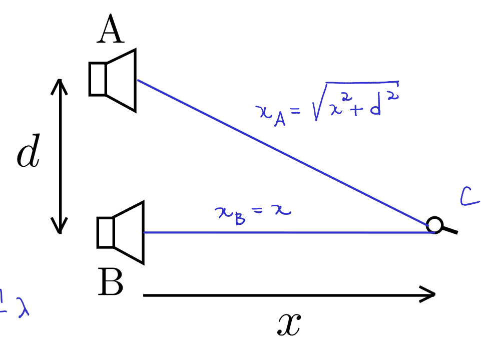


Figure 1: Interference of loudspeaker sound waves.

phase difference $\rightarrow$ $\Delta\varphi_{A,B}$ at C $= \varphi_A - \varphi_B = k(x_A - x_B) = \frac{2\pi}{\lambda}(x_A - x_B)$

a) $\Delta\varphi = (2n-1)\pi,\ n \in \mathbb{N}$ or $x_A - x_B = \sqrt{x^2 + d^2} - x = \frac{2n-1}{2}\lambda$

$\Rightarrow x_{\text{des.}} = \lambda\left(\frac{1}{2n-1}\frac{d^2}{\lambda^2} - \frac{2n-1}{4}\right)$

$\Delta\varphi > 0$ because $x_A > x_B$

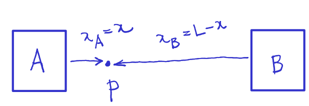
b) $\Delta\varphi = 2n\pi \rightarrow x_{\text{con.}} = \lambda\left(\frac{1}{2n}\frac{d^2}{\lambda^2} - \frac{n}{2}\right)$.

c) $n = 1 \rightarrow x_{\text{des.}} = \frac{d^2}{\lambda} - \frac{\lambda}{4} < 0 \rightarrow \lambda > 2d$.

4) Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is $L = 2.00$ m to the right of speaker A. The frequency of the sound waves produced by the loudspeakers is $f = 206$ Hz. Consider point P between the speakers and along the line connecting them, a distance x to the right of speaker A. Both speakers emit sound waves that travel directly from the speaker to point P.
a) For what values of x will destructive interference occur at point P?
b) For what values of x will constructive interference occur at point P?
c) Interference effects like those in parts 'a' and 'b' are almost never a factor in listening to home stereo equipment. Why not?

a) $x_A - x_B = x - (L - x) = 2x - L = \frac{2l-1}{2}\lambda,\quad l \in \mathbb{Z}$

$\Rightarrow x = \frac{L}{2} + \frac{2l-1}{4}\lambda$.



b) $x_A - x_B = 2x - L = l\lambda \Rightarrow x = \frac{L}{2} + \frac{l}{2}\lambda$.

c) reflections