dynamics of rotational motion, problem set #2

1) A uniform spherical boulder with mass $m = 1$ kg starts from rest and rolls down a $H = 50$ m high hill, as shown in fig. 1. The top half of the hill is rough enough to cause the boulder to roll without slipping, but the lower half is covered with ice and the boulder comes down sliding.

a) What is the total mechanical energy at the beginning? Define zero potential height to be the ground down the hill.
b) What is the total mechanical energy half-way at height $h = 25$ m?
c) Write an equation for kinetic energy of the boulder at height $h = 25$ m. Find the velocity v_h and angular velocity ω_h at this height.
d) Will angular velocity change coming down on the smooth part, if there is no friction between the boulder and the ice surface? What is the translational speed of the boulder when it reaches the bottom of the hill?

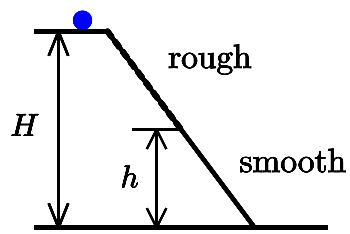

Figure 1: A boulder rolling down the hill.

a) $E = mgH$

b) E is conserved $\rightarrow$ $E = mgH$

c) it's still rolling without slipping $\rightarrow$ $\omega_h = \frac{v_h}{r}$

$$mgH = mgh + \frac{1}{2} m v_h^2 + \frac{1}{2} I \omega_h^2$$
$$= mgh + \frac{1}{2} m v_h^2 + \frac{1}{2}\left(\frac{2}{5} m r^2\right) \omega_h^2$$
$$= mgh + \frac{7}{10} m v_h^2$$

$$\rightarrow v_h = \sqrt{\frac{10}{7} g (H-h)}$$

$$\frac{5}{7} H + \frac{2}{7} h$$

d) no, the torque with respect to center of sphere is 0.

again we can use conservation of energy:

$$mgH = \frac{1}{2} m v^2 + \frac{1}{2} I \omega_h^2$$
$$= \frac{1}{2} m v^2 + \frac{2}{7} mg (H-h)$$

$$\rightarrow v = \sqrt{2gh + v_h^2} = \sqrt{g\left(\frac{10}{7} H + \frac{4}{7} h\right)}$$

2) Two masses $m_1 = 1.0$ kg and $m_2 = 0.5$ kg are connected by a very light, flexible cord that passes over a frictionless pulley of mass $M = 0.5$ kg and radius $R = 0.3$ m. The pulley is a solid uniform disk and is supported by a hook connected to the ceiling as shown in fig. 2. Define the positive direction for the acceleration a and angular acceleration of the pulley α to be as shown in the figure. We call the tension forces T, T_1, and T_2. As you guessed, $T = T_1 + T_2$ because the pulley is not moving.

a) Write the equation of motion for the two masses m_1 and m_2. Find $T_1 - T_2$ in terms of a.
b) Why $T_1 \neq T_2$?
c) Write the equation of rotational dynamics for the pulley.
d) Remember there is a relation between a and α. Using part 'a' and 'c', find a.

Now we will try another method. Recall that if you have a system then $\tau_{\text{ext}} = \mathrm{d}L_{\text{tot}}/\mathrm{d}t$. Call the center of the pulley point O.
e) Find τ_{ext}, the external torque acting on the system of pulley and the masses with respect to point O. Remember the tension forces T_1 and T_2 count as internal forces now, and T does not make any contribution.
f) Write angular momentum of the system with respect to O in terms of given parameters and v, the velocity of the masses.
g) Now find $a = \mathrm{d}v/\mathrm{d}t$.

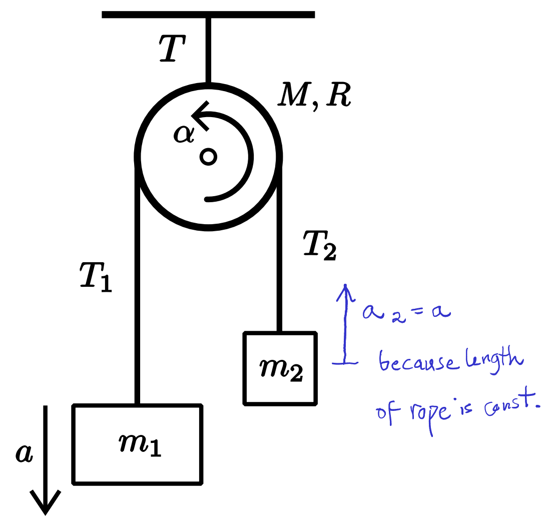

Figure 2: Two mass connected over a pulley.

a) $\begin{cases} m_1 g - T_1 = m_1 a \\ T_2 - m_2 g = m_2 a \end{cases} \rightarrow T_1 - T_2 = (m_1 - m_2)g - (m_1 + m_2)a \quad (1)$

b) $m_1 \neq m_2$ and pulley is rotating, and $M \neq 0$, if $T_1 = T_2 \rightarrow \alpha = 0$, which is not the case.

c) $\sum \tau = I\alpha \rightarrow RT_1 - RT_2 = I\alpha \rightarrow R(T_1 - T_2) = \frac{1}{2}MR^2\alpha. \quad (2)$

d) there is no slipping so $\alpha = \frac{a}{R}$, putting (1) in (2): $a = \frac{(m_1 - m_2)g}{(m_1 + m_2 + \frac{1}{2}M)}$.

e) Only ext. forces are T and $m_1 g$ and $m_2 g$. T has no contribution, and the lever arm for both $m_1 g$ and $m_2 g$ is R. $\tau_{ext} = Rg(m_1 - m_2)$.

f) We have $+I\omega$ of the pulley, and both $p_1 = m_1 v$ and $p_2 = m_2 v$ rotating CCW, w/ lever arm R, so $L_{tot} = I\frac{v}{R} + R(m_1 + m_2)v = \frac{1}{2}MRv + R(m_1 + m_2)v$

g) $\tau_{ext} = \frac{dL_{tot}}{dt} \rightarrow Rg(m_1 - m_2) = \frac{dv}{dt}\left[\frac{1}{2}MR + R(m_1 + m_2)\right] \rightarrow a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{1}{2}M}$.