equilibrium and elasticity, problem set #2

1) A uniform rod with length $L = 4$ m and mass $M = 6$ kg is sitting on a fulcrum, and it is in equilibrium with two other masses, $m_1 = 12$kg and m_2.

a) When m_1 sitting at one end of the rod, and m_2 is siting $l = 1$ m from the other end, the system is in equilibrium if fulcrum is $l_f = 1$ m from m_1, as shown in fig. 1. Find m_2.
b) Now if both masses m_1 and m_2 sit on two ends of the rod, where must we put fulcrum so that the system is in equilibrium?

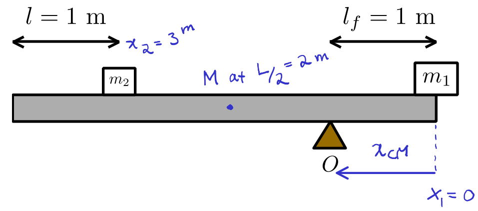
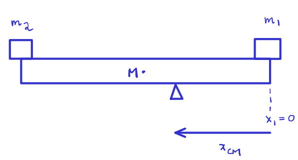


Figure 1: The rod with two masses, for part 'a'.

a) $x_{CM} = 1^{m} = \dfrac{m_1 x_1 + m_2 x_2 + M\,L/2}{m_1 + m_2 + M} = \dfrac{0 + 3m_2 + 12}{m_2 + 18} \Rightarrow m_2 = 3\text{ kg}.$

b) $x_{CM} = \dfrac{m_1 x_1 + m_2 x_2 + M\,L/2}{m_1 + m_2 + M}$

$= \dfrac{0 + 3\times 4 + 12}{12 + 3 + 6} = \dfrac{8}{7}\,m$



2) Consider a uniform beam with length L and math M, is hinged to a wall and supported by a horizontal rope, as shown in fig. 2. The angle between the beam and horizontal line is $\theta = 30°$.

a) Draw the free body diagram for the beam.
b) Find the tension force T of the rope.
c) Rod is pushing the hinge. Find this force's direction and magnitude.

a)

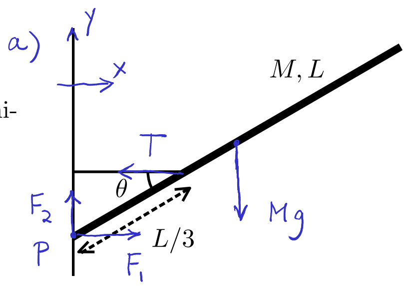


Figure 2: A beam connected to wall.

b)

$$\sum \vec{F} = 0 \quad \begin{cases} \hat{x}: & F_1 - T = 0 \quad (1) \\ \hat{y}: & F_2 - Mg = 0 \quad (2) \end{cases}$$

$$\sum \tau_P = 0 \ : \quad T \frac{L}{3} \sin\theta - Mg \frac{L}{2} \cos\theta = 0 \quad (3)$$

$$(3): \quad T = \frac{3}{2} Mg \cot\theta$$

c)

$$\left.\begin{aligned} (1): \quad & F_1 = T = \frac{3}{2} Mg \cot\theta \\ (2): \quad & F_2 = Mg \end{aligned}\right\} \Rightarrow \quad \begin{aligned} F &= Mg \sqrt{1 + \frac{9}{4} \cot^2\theta} \\ \hat{F} &= \frac{\hat{x}\, 3/2 \cot\theta + \hat{y}}{\sqrt{1 + 9/4 \cot^2\theta}} \end{aligned}$$

3) Two uniform, $m = 50$ gr marbles, with radius $r = 1.0$ cm are stacked as shown in fig. 3 in a container that is $w = 3.0$ cm wide.

a) Find the angle θ between the horizontal line and the line connecting the centers.
b) Draw the free body diagram for each marble. Call the forces at the points A, B, and C as N_A, N_B, and N_C, respectively.
c) Consider two marbles as a system and explain why $N_A = N_C$ and find N_B.
d) Now considering equilibrium of any one of the marbles and find N_A or N_C.
e) The length of the container is L and we apply a force F horizontally to the top edge of the container. The container's mass is negligible and it does not slide on the ground. Find minimum of F to knock over the system.

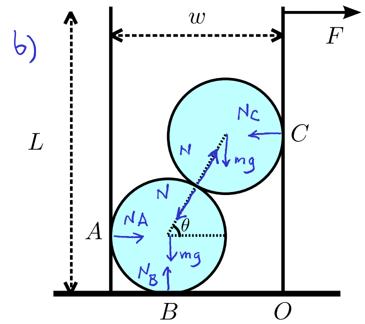


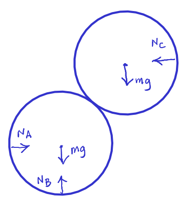
Figure 3: The marbles inside a container.

a) $W = R + 2R\cos\theta + R$

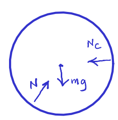
$\cos\theta = \frac{1}{2} \rightarrow \theta = \pi/6$

c) N is internal.

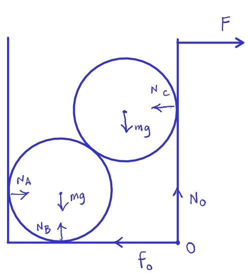
$\sum \vec{F} = 0 \begin{cases} \hat{x}: & N_A - N_C = 0 \rightarrow N_A = N_C \\ \hat{y}: & N_B - mg - mg = 0 \rightarrow N_B = 2mg \end{cases}$



d) $\sum \vec{F} = 0 \begin{cases} N\cos\theta = N_C \\ N\sin\theta = mg \end{cases} \rightarrow N_C = mg\cot\theta$



e) if you choose container + marbles as the system the only ext. forces are F, two marble weights, and the forces at point O, N_o and f_o.



$\tau_o = 0 \; : \; -FL + mgR + mg(2R) = 0$

$\Rightarrow F = \frac{3R}{L} mg.$