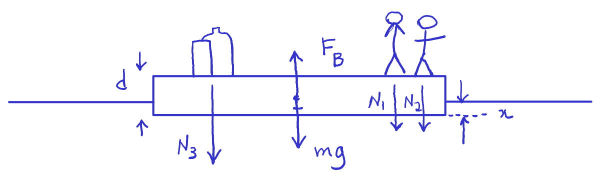
fluid mechanics, problem set #2

1) A solid, square pinewood raft measures $l = 5$ m on a side and is $d = 0.40$ m thick, and $\rho_{\text{pine}} = 550\ \text{kg/m}^3$. Two persons and some diving gears are on top of the raft with total mass of $M = 700$ kg.
a) Find the weight of only the raft.
b) Draw all the forces acting on the raft.
c) Write the equilibrium equation and find the buoyant force.
d) How much of the raft is beneath the surface of water?
e) What is the maximum weight for gears and people we can have on the raft before it goes completely under water?

a) $W_{raft} = mg = \rho_{pine} V_{raft}\, g = \rho_{pine} \ell^2 d\, g = 5.5 \times 10^4$ N

b) $N_1 + N_2 + N_3 = Mg = 7.0 \times 10^3$ N



c) $Mg + mg - F_B = 0$

$\Rightarrow F_B = (M+m)g = 6.2 \times 10^4$ N

d) $F_B = \rho_{water}\, \ell^2 x\, g = 2.5 \times 10^5\ x/m$ N , using part 'c' $x = 0.25$ m

e) $\text{Max}\left[F_B\right] = F_B\Big|_{x=d} = \rho_{water}\, \ell^2 d\, g = 1.0 \times 10^5$ N

$M_{max}\, g = F_{B\,max} - mg \Rightarrow M_{max} = \rho_{water}\, \ell^2 d - m = 4.5 \times 10^3$ kg .

2) A cubical block of wood with side length $l = 10$ cm is sitting at the interface of water and oil in equilibrium as shown in fig. 1. The total oil height is $h = 12$ cm and the height of block inside water is $l_w = 2$ cm. Also call the densities of water and oil, $\rho_{\text{w}} = 1.0 \times 10^3$ kg/m^3 and $\rho_{\text{o}} = 0.64 \times 10^3$ kg/m^3, respectively. We want to find the density of the wood.

a) Find the pressure inside the oil at point Q.
b) Find the force of oil on the top surface of the block.
c) Find the pressure at point R inside the water.
d) Find the force of water on the bottom of the block.
e) Write the equilibrium condition for the block and find the density of the wood, ρ_{wood}.

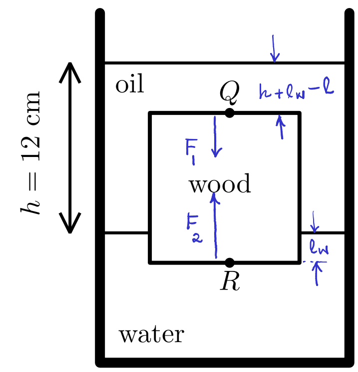


Figure 1: A cubic wooden block at the interface.

a) $P_Q = P_0 + \rho_0 g\,(h + l_w - l)$

b) $F_1 = \left[P_0 + \rho_0\, g\,(h + l_w - l)\right] l^2$

c) $P_R = P_0 + \rho_0 g h + \rho_w g\, l_w$

d) $F_2 = \left[P_0 + \rho_0 g h + \rho_w g\, l_w\right] l^2$

e) $F_2 - F_1 - \rho_{wood}\, l^3 g = 0$

$$\Rightarrow \boxed{\rho_{wood} = \rho_0 + \frac{l_w}{l}\left(\rho_w - \rho_0\right)}$$

Another way is to use Archimedes law, $\begin{cases} l_w l^2 & \text{volume in water} \\ (l - l_w)\, l^2 & \text{" " oil} \end{cases}$

$\rho_{wood}\, l^3 g - \rho_w g\left[l_w l^2\right] - \rho_0 g\left[(l - l_w)\, l^2\right] = 0$

equilibrium for water and oil if block was not there