mechanical waves, problem set #2

1) Consider an irregular beam with weight $W = 1.0 \times 10^3$ N, hanging horizontally using two similar strings A and B, with the same length $l = 3.0$ m and mass $m = 0.20$ kg, as shown in the fig. 1. The center of gravity of the beam is one-fifth of the way along the beam from the string A. Call the tensions in the strings A and B, T_A and T_B, respectively. These tensions are constant through the strings.

a) Write down the force equilibrium condition in vertical direction, $\sum F_y = 0$, for the beam.
b) Write down the rotational equilibrium condition, $\sum \tau_O = 0$, for the beam.
c) Using previous parts, find T_A and T_B.
d) We pluck both strings at the same time at the bottom. Find the time difference that the pulses reaches the ceiling in both strings. Which pulse arrives first?
e) We pluck both strings again this time causing them to vibrate in their 3rd overtone. Find the frequencies of the sound waves that strings A and B produce.

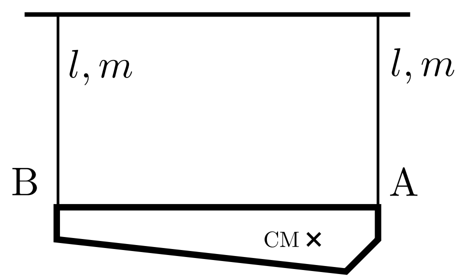
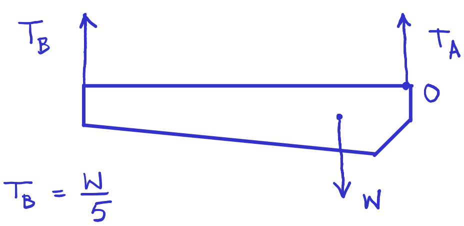


Figure 1: A beam hanged by two strings.

a) $T_A + T_B = W$

b) I choose O to be the bottom of the string A



beam length

$$W \frac{L}{5} - T_B L = 0 \Rightarrow T_B = \frac{W}{5}$$

c) $T_B = \frac{W}{5} = 2.0 \times 10^2$ N, $\quad T_A = \frac{4}{5} W = 8.0 \times 10^2$ N.

d) $v_A = \sqrt{\frac{T_A}{\mu}} = 110 \frac{m}{s}$, $\quad v_B = \sqrt{\frac{T_B}{\mu}} = 55 \frac{m}{s}$ $\qquad t_A = \frac{l}{v_A} = 27$ ms, $\quad t_B = \frac{l}{v_B} = 54$ ms

pulse in A arrives first, pulse in B takes twice as long.

e) $\lambda = \frac{2l}{3} = 2.0$ m

for both strings

$f_A = v_A / \lambda = 55$ Hz $= f_{s_1}$

$f_B = v_B / \lambda = 28$ Hz $= f_{s_2}$

produced sound freq's are the same as source freq's, f_A and f_B.

2) A wire with length $l = 10$ m and mass $m = 0.20$ kg is supported by two uniform poles with masses $M = 10$ kg, as shown in fig. 2. The poles are hinged to the ground with no friction. The angle between poles and ground is $\theta = 45°$. T is the tension force in the wire.

a) Write down the rotational equilibrium condition, $\sum \tau_O = 0$, for a pole and find T. Neglect the mass of the wire for this part and assume that the wire is horizontal at all points.

b) We pluck the wire at one end. How long it takes for the pulse to travel to the other end?

c) Wind causes the wire to vibrate in its 4th overtone. Find the frequency and wavelength of the sound wave that the wire would produce. Take the sound velocity of the air to be $v_s = 340$ m/s.

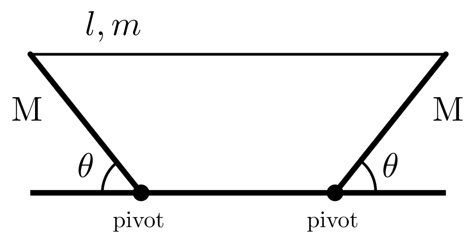
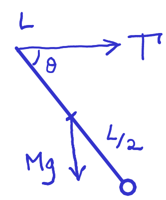


Figure 2: A wire supported by two poles.

a) $$Mg\frac{L}{2}\cos\theta - TL\sin\theta = 0$$

$$\Rightarrow \quad T = \frac{1}{2}Mg\cot\theta = 50^{N}.$$



b) $$v = \sqrt{\frac{T}{\mu}} \qquad \mu = \frac{m}{l} = \frac{0.20\text{ kg}}{10\text{ m}} = .020\,\frac{\text{kg}}{\text{m}}$$

$$v = \sqrt{\frac{50^{N}}{.020\,\frac{\text{kg}}{\text{m}}}} = 50\ \text{m/s}$$

$$t = \frac{l}{v} = \frac{10^{m}}{50\ \text{m/s}} = 0.20\ \text{s}.$$

c) $$\lambda = \frac{2l}{n} = \frac{20^{m}}{4} = 5\ \text{m} \quad \Rightarrow \quad f = \frac{v}{\lambda} = \frac{50\ \text{m/s}}{5\ \text{m}} = 10\ \text{Hz}$$

frequency of the sound is the same

$$f_s = f = 10\ \text{Hz}$$

$$\lambda_s = \frac{v_s}{f_s} = \frac{340\ \text{m/s}}{10\ \text{Hz}} = 34\ \text{m}.$$