simple harmonic motion, problem set #2

1) Consider a horizontal tray with mass M which is attached to an ideal vertical spring with stiffness k. There is a brick with mass m sitting on this tray. See fig. 1. We push down the tray to $y = -A$, where $y = 0$ is the equilibrium point. We release the tray with zero velocity, $v = 0$. In other words $y(0) = -A$ and $v(0) = 0$ are the initial conditions. If A is large enough the brick will leave the tray.

a) What is the angular frequency ω when brick is on the tray?
b) Write down $y(t)$, assuming brick is on the tray.
c) Now write down $a(t)$, the acceleration of the system.
d) Draw the free body diagram for the brick. We call the normal force N.
e) Write down the equation of motion for the brick and find $N(t)$, normal force as a function of time.
f) What is the condition on N when brick leaves the tray?
g) Find minimum A so that above condition has a solution, i.e. brick leaves the tray at some point. Find when and where the brick leaves the tray.

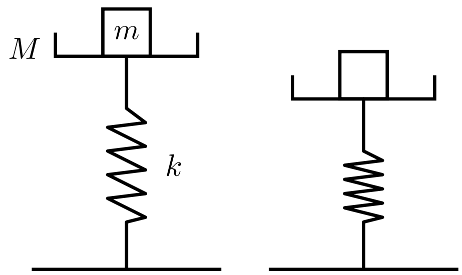


Figure 1: left: equilibrium, right: initial condition

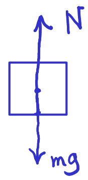
a) the total mass is $m + M$ $\rightarrow$ $\omega = \sqrt{\dfrac{k}{m+M}}$.

b) $y(t) = A\cos(\omega t + \varphi_0)$ initial con. $y(0) = -A$, $v(0) = 0$ $\longrightarrow$ $\varphi_0 = \pi$

$y(t) = -A\cos(\omega t)$.

c) $v(t) = \dfrac{d}{dt}y(t) = A\omega\sin(\omega t)$, $a(t) = \dfrac{d}{dt}v(t) = A\omega^2\cos(\omega t)$

d)



e) $N - mg = m a(t) \Rightarrow N(t) = m[g + a(t)] = m\left[g + A\omega^2\cos(\omega t)\right]$.

f) normal force becomes zero right before brick leaves the tray, $N(t = t_1) = 0$.

g) $N = 0 \longrightarrow g + A\omega^2\cos(\omega t_1) = 0 \longrightarrow \cos(\omega t_1) = -\dfrac{g}{A\omega^2}$

$|\cos(\omega t)| \leq 1 \Rightarrow A_{min} = \dfrac{g}{\omega^2}$, if $A = A_{min}$ right at the top point brick almost looses the touch w/ tray.

$t_1 = \cos^{-1}\left(-\dfrac{g}{A\omega^2}\right)$, $y(t_1) = -A\cos(\omega t_1) = -A\left(\dfrac{-g}{A\omega^2}\right) = \dfrac{g}{\omega^2}$.

2) A mass M is attached to a horizontal spring with stiffness k as shown in fig. 2. A block with mass m is sitting on top of it and there is friction between two masses. Take μ_S to be static friction coefficient. We pull the system of masses to point $x = A$ and release it; in other words initial conditions are $x(0) = A$ and $v(0) = 0$. If the amplitude A is large enough the block on top will slide. In all parts we are discussing the time before the block slides, if it does.

a) What is the angular frequency ω?
b) Write down $x(t)$.
c) Write down $a(t)$, the acceleration of the system.
d) Draw the free body diagram for the block on top.
e) Write down the equation of motion for the block on top and find $f_S(t)$, static friction force as a function of time.
f) What is the condition so that the block on top slides at some point in time? Find when and where it happens.

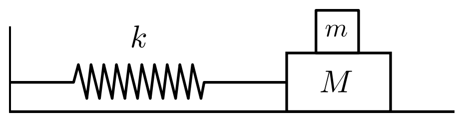


Figure 2: If the amplitude is large enough the block on top will slide.

a) $\omega = \sqrt{\dfrac{k_{eff}}{m_{eff}}} = \sqrt{\dfrac{k}{m+M}}$

b) $x(t) = \mathcal{A}\cos(\omega t + \varphi_0)$ $\qquad v(0) = -\mathcal{A}\omega\sin\varphi_0 = 0 \rightarrow \varphi_0 = 0.$

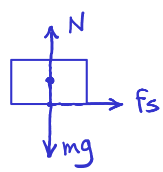
general formula $\qquad x(0) = \mathcal{A}\cos\varphi_0 = A \rightarrow \mathcal{A} = A.$

$x(t) = A\cos(\omega t).$

c) either $a(t) = -\mathcal{A}\omega^2\cos(\omega t + \varphi_0) = -A\omega^2\cos(\omega t)$

or do the derivatives yourself: $a(t) = \dfrac{d^2}{dt^2}\left(A\cos(\omega t)\right) = -A\omega^2\cos(\omega t).$

d)



e) $\sum \vec{F} = m\vec{a}$
$\hat{x}$: $f_s = ma = -mA\omega^2\cos(\omega t)$
$\hat{y}$: $N - mg = 0 \Rightarrow N = mg$

$<$: not sliding
$=$: almost not sliding

f) $f_s \le \mu_s N = \mu_s mg \Rightarrow -mA\omega^2\cos(\omega t) \le \mu_s mg$

$\Rightarrow A_{min} = \dfrac{\mu_s g}{\omega^2}$ and

for $A > A_{min}$ we have a solution for t^*: $t^* = \dfrac{1}{\omega}\cos^{-1}\left(\dfrac{\mu_s g}{\omega^2}\right)$

the point it starts sliding : $x(t^*) = A\cos(\omega t^*) = \dfrac{\mu_s g}{\omega^2} A.$