temperature and heat, problem set #2

1) An aluminum tea kettle with mass $m_{\mathrm{Al}} = 1.5$ kg and containing $m_{\mathrm{W}} = 1.8$ kg of water is placed on a stove. The specific heat capacity of water, ice, and aluminum are $c_{\mathrm{W}} = 4.2$ kJ/kg $\cdot$ K, $c_{\mathrm{I}} = 2.1$ kJ/kg $\cdot$ K, and $c_{\mathrm{Al}} = 0.91$ kJ/kg $\cdot$ K, respectively. The latent heat of fusion for water is $L_{\mathrm{f}} = 334$ kJ/kg.
a) If no heat is lost to the surrounding, how much heat must be added to raise the temperature from 20°C to 85°C?
b) Now if we throw a piece of ice with mass $m_{\mathrm{I}} = 0.5$ kg and temperature -20°C inside this hot water, what would be the final temperature of the system?

a) $-Q + m_W c_W (85^\circ C - 20^\circ C) + m_{Al} c_{Al} (85^\circ C - 20^\circ C) = 0 \rightarrow Q = 5.8 \times 10^5$ J.

b) $m_W c_W (T - 85^\circ C) + m_{Al} c_{Al} (T - 85^\circ C) + m_I c_I (0^\circ C - (-20^\circ C)) + m_I L_f + m_I c_W (T - 0^\circ C) = 0$

$\rightarrow T = 52^\circ C$.

if you found that $T < 0$: write down equation above with $T = 0^\circ C$ and $m L_f$ instead of $m_I L_f$ where $m < m_I$ is mass of the ice which is melted. So $T = 0^\circ C$ is final equilibrium temperature w/ $m_I - m$ ice remained.

2) Consider an aluminum bar with $\alpha_1 = 2.4 \times 10^{-5}$ /K and $Y_1 = 70$ GPa, and a steel bar with $\alpha_2 = 1.2 \times 10^{-5}$ /K and $Y_2 = 210$ GPa. As shown in fig. 1 these two bars are sitting between two solid walls with a gap $D = 1.96$ m. If there is no tension the length of the bars are $L = 1$ m at 20°C and their area is $A = 1.0 \times 10^{-3}$ m^2. We call the joint point position x, so the aluminum bar's length is x and the steel bar's length is $D - x = 1.96$ m $- x$. Remember the force formula for a change in length ΔL and a change in temperature ΔT is given by, $F = YA(\Delta L/L_0 - \alpha \Delta T)$. As $\alpha \Delta T \ll 1$ and $\Delta L \ll L_0$, use the same number for A which is given. The walls will not move if temperature changes, in other words, D is constant.

a) Use $\Delta T = 0$ and write down mechanical equilibrium condition for the bars and find x at 20°C.
b) Write the equilibrium condition at temperature $T = 400$°C and find x at 400°C.

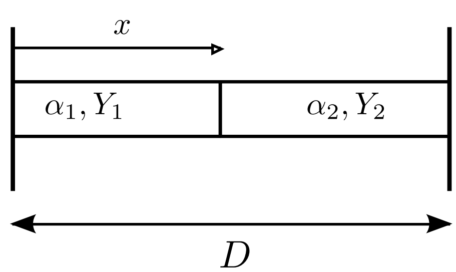

Figure 1: Bars between two solid walls.

a) $F_1 = F_2$

$\rightarrow Y_1 \cancel{A} \left(\frac{L_1 - L_0}{\cancel{L_0}} \right) = Y_2 \cancel{A} \left(\frac{L_2 - L_0}{\cancel{L_0}} \right)$

$\rightarrow Y_1 (x - L_0) = Y_2 (D - x - L_0)$

$\rightarrow x = \frac{(Y_1 - Y_2) L_0 + Y_2 D}{Y_1 + Y_2} = 97^{cm}$.

& $D - x = 99$ cm for steel bar.

b) $F_1 = F_2$

$\rightarrow Y_1 \left(\frac{x - L_0}{L_0} - \alpha_1 \Delta T \right) = Y_2 \left(\frac{D - x - L_0}{L_0} - \alpha_2 \Delta T \right)$

$\rightarrow x = \frac{Y_1 L_0 (1 + \alpha_1 \Delta T) - Y_2 L_0 (1 + \alpha_2 \Delta T) + Y_2 D}{Y_1 + Y_2}$

$= 97^{cm} + \frac{100\,cm \cdot \alpha_1 \Delta T - 300\,cm \cdot \alpha_2 \Delta T}{4} = 96.9$ cm.

formula $F = YA\left(\frac{\overbrace{\Delta L}^{L - L_0}}{L_0} - \alpha \Delta T\right)$

L: final length changed due to the pressure and temperature change

$\Delta L = \frac{F L_0}{A Y} + \alpha L_0 \Delta T$