thermal properties, problem set #2

1) Estimate the number of atoms in the body of a 50 kg physics student. Note that the human body is mostly water, which has molar mass 18.0 g/mol, and that each water molecule contains three atoms.

$$n = \frac{50 \times 10^3 \text{ g}}{18.0 \text{ g/mol}} = 2.8 \times 10^3 \text{ mol} \rightarrow N = nN_A = 2.8 \times 10^3 \text{ mol} \times 6.0 \times 10^{23} \frac{\text{molecules}}{\text{mol}} = 1.7 \times 10^{26} \text{ molecules.}$$

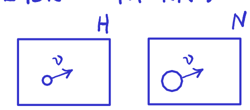
$$\rightarrow N_{atoms} = 3N = 5 \times 10^{26} \text{ atoms.}$$

2)
a) At what temperature is the root-mean-square speed of nitrogen atom equal to the root-mean-square speed of hydrogen atom at 20°C? *Hint: Be careful about the temperature unit and* $m_N = 14m_H$.
b) If we fill two balloons of the same volume, one with 1 mol of hydrogen at 20°C and the other with 1 mol of nitrogen at the temperature you found in part part a, (so that the molecules of both have the same root-mean-square speed), compare the balloons' pressures.

← in practice it's molecules but again, $m_{N_2} = 14 m_{H_2}$.

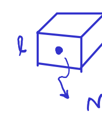
a) $\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT \rightarrow$ same v_{rms} : $T \propto m \rightarrow T_N = \frac{m_N}{m_H} T_H = 14 \times 293\text{K} = 4.1 \text{ kK}.$

b) $P_H = \frac{1 \text{ mol} \cdot R \cdot 293\text{K}}{V}$, $P_N = \frac{1 \text{ mol} \cdot R \cdot 4.1 \text{ kK}}{V}$ So $\frac{P_N}{P_H} = 14.$



3) Consider an ideal gas at 27°C and 1.00 atm pressure. To get some idea how close these molecules are to each other, on the average, imagine them to be uniformly spaced, with each molecule at the center of a small cube.
a) What is the length of an edge of each cube if adjacent cubes touch but do not overlap?
b) How does their separation compare with the spacing of atoms in solids, which typically are about 0.3 nm apart?

a) $N = 1$ for each cube, and $V = \ell^3$, where ℓ is the length of an edge.
$PV = NkT \rightarrow 10^5 \ell^3 = 1 \times 1.38 \times 10^{-23} \times 300 \rightarrow \ell = 3.5 \text{ nm}.$



b) $\ell \gg 0.3$ nm, about one order of magnitude.

4) A large tank of water has a hose connected to it, as shown in fig. 1. The tank is sealed at the top and has compressed air between the water surface and the top. When the water height h has the value 3.50 m, the absolute pressure p of the compressed air is 4.00×10^5 Pa. Assume that the air above the water expands at constant temperature, and take the atmospheric pressure to be 1.00×10^5 Pa.

$v_1 A_1 = v_2 A_2$
$v_1 = \frac{A_2}{A_1} v_2 \approx 0$
$\frac{A_2}{A_1} \ll 1$

a) What is the speed with which water flows out of the hose when $h = 3.50$ m.
b) As water flows out of the tank, h decreases. Calculate the speed of flow for any h and determine when the flow stops.

a) $P_1 + \rho g h + \frac{1}{2}\rho \cancel{v_1^2} = P_2 + \rho g \times 0 + \frac{1}{2}\rho v_2^2$ ($v_1^2 \approx 0$)

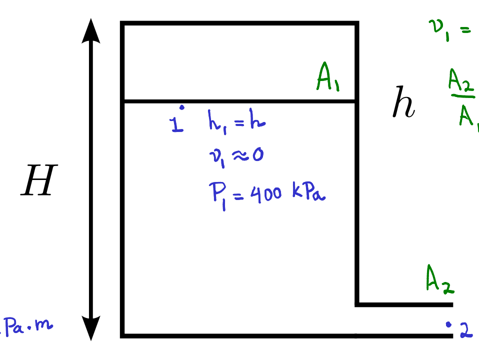
$v_2 = \sqrt{2\left[gh + (P_1 - P_2)/\rho\right]} = 26 \text{ m/s}$

b) $P_1 V = nRT$ and $V = A_1(H-h)$

$\Rightarrow P_1(H-h) = \frac{nRT}{A_1} = \text{const.} = (400 \text{ kPa})(0.50 \text{ m}) = 200 \text{ kPa}\cdot\text{m}$

So $v_2(h) = \sqrt{2\left[gh + \left(\frac{200 \text{ kPa}\cdot\text{m}}{H-h} - 100 \text{ kPa}\right)/\rho\right.}$

$= \sqrt{20\left[-10 + h + \frac{20}{4-h}\right]} \text{ m/s}.$



$h_2 = 0$
$P_2 = 100$ kPa
$v_2 = ?$

Figure 1: A tank with compressed air on top.

$v_2(h) = 0 \rightarrow h = 1.6 \text{ m}.$