waves, problem set #2

1) A pipe closed at one end, playing in the fundamental, resonates with a string with length l, mass m, and tension T, in 4th overtone. Call sound velocity in the air v_{air}. If we open both ends of the same pipe, and play it in its 3rd overtone what is the wavelength and frequency of the sound wave it produces?

L — 1st or fundamental: $\lambda = 4L$

$f_{pipe} = f_{String}$

m, ℓ, T, 4th overtone: $v = \sqrt{\frac{T\ell}{m}}$, $\lambda = \frac{2\ell}{4} = \frac{\ell}{2}$

$\frac{v_{air}}{4L} = \frac{\sqrt{T\ell/m}}{\ell/2} \Rightarrow L = \frac{1}{8} v_{air}\sqrt{\frac{m\ell}{T}}$.

Open pipe, L: $\lambda = \frac{2L}{n} = \frac{2L}{3} = \frac{1}{12} v_{air}\sqrt{\frac{m\ell}{T}}$, $f = \frac{v_{air}}{\lambda} = 12\sqrt{\frac{T}{m\ell}}$.

2) A wire has length 0.50 m and mass 0.020 kg. A transverse wave in the wire is described by $y(x,t) = 0.2\cos(2x - 5t)$ where y and x are in meters, and t in seconds.
a) What is the tension on the wire?
b) If we fix both ends and make standing waves in this wire with same tension, what is the fundamental resonance frequency of the string?
c) What is the resonance frequency if there are two nodes, in addition to the two nodes at the end?

a) $y(x,t) = 0.2\cos(2x-5t)$ $k = 2\ \frac{rad}{m}$, $\omega = 5\ \frac{rad}{s}$ $\rightarrow v = \frac{\omega}{k} = 2.5\ m/s = \sqrt{\frac{T}{m/\ell}}$
$\hookrightarrow T = \frac{m}{\ell}v^2 = 0.040\ \frac{kg}{m} \times (2.5\ m/s)^2 = 0.25\ N$.

b) $\lambda = \frac{2\ell}{1} = 1.0\ m \rightarrow f = \frac{v}{\lambda} = \frac{2.5\ m/s}{1.0\ m} = 2.5\ Hz$.

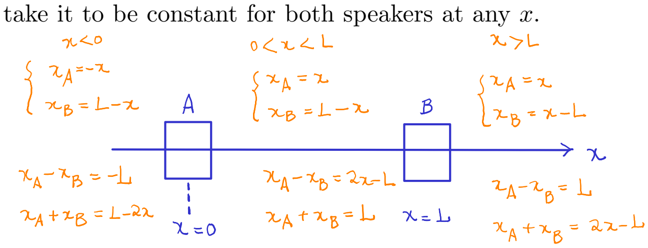
c) if you add two more nodes  it will be 3rd overtone $\rightarrow \lambda = \frac{2\ell}{3} = .33\ m \rightarrow f = \frac{2.5}{.33} = 7.5\ Hz$.

3) Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase with amplitude A. Speaker B is $L = 2.0$ m to the right of speaker A. The frequency of the sound waves produced by the loudspeakers is $f = 440$ Hz. Consider point P along the line connecting them at a distance x to the right of speaker A. Take all three cases, $x < 0$, $0 < x < L$, and $L < x$. Both speakers emit sound waves that travel directly from the speaker to point P. Although the amplitude of the wave is inversely proportional to distance from source, but take it to be constant for both speakers at any x. Find the interference pattern (the amplitude) at x.

At any point P the total wave amplitude is sum of amplitudes of speaker A & B,

$y(x,t) = Y_A + Y_B = A\sin\varphi_A + A\sin\varphi_B$

where A is the amplitude and φ_A, φ_B are the phases.



Take $\varphi_A = 0$ for $x = 0$, $\varphi_B = 0$ for $x = L$: when sound comes out of speakers phase = 0.

$\rightarrow y(x,t) = A\sin(kx_A - \omega t) + A\sin(kx_B - \omega t)$ (distance traveled from A; going away from speaker; distance traveled from B)

$= 2A\cos\left(\frac{k(x_A - x_B)}{2}\right)\sin\left(\frac{k(x_A+x_B)}{2} - \omega t\right)$

$2A\cos\left(\frac{k(x_A - x_B)}{2}\right)$: at any x this can be thought about as amplitude (no time dependence)

$$y(x,t) = 2A\begin{cases}\cos\left(-\frac{kL}{2}\right)\sin\left(\frac{kL}{2} - kx - \omega t\right) & x<0 \\ \cos\left[k\left(x - \frac{L}{2}\right)\right]\sin\left(\frac{kL}{2} - \omega t\right) & 0<x<L \\ \cos\left(\frac{kL}{2}\right)\sin\left(-\frac{kL}{2} + kx - \omega t\right) & x>L\end{cases}$$

$x<0$: left traveling wave with amplitude $2A\cos\left(-\frac{kL}{2}\right)$

$0<x<L$: Standing wave w/ antinode at $x = \frac{L}{2}$

$x>L$: right traveling wave with amplitude $2A\cos\left(\frac{kL}{2}\right)$